1. (20%)

It is known that $\mathbf{AB}=\mathbf{I}$, and $\mathbf{A}=\mathbf{LDL}^t$, where **A** and **B** are 5×5 matrices, **I** is a 5×5 unit matrix, $\mathbf{L}^t$ is the transpose of matrix **L**. It is known that.

$$\mathbf{L}=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&-2&1&0&0\\0&-3&0&1&0\\0&-4&0&0&1\end{bmatrix},\quad \mathbf{D}=\begin{bmatrix}1&0&0&0&0\\0&2&0&0&0\\0&0&4&0&0\\0&0&0&2&0\\0&0&0&0&1\end{bmatrix}.$$

Please determine the inverse of **L** and the values of entries B_{2j} (j = 1-5) of matrix **B**.

2. (20%)

Let $F(x)$ be the function defined by

$$F(x)=\sum_{n=-\infty}^{n=\infty}\cosh(x-2nL)\left[u\left(x-(2n-1)L\right)-u\left(x-(2n+1)L\right)\right]$$

where L is a constant, u is the unit step function defined by

$$u(x-a)=\begin{cases}0 & \text{for } x<a\\1 & \text{for } x>a\end{cases}$$

(a) Determine $F'(x)$, the derivative of $F(x)$ with respect to x.

(b) Sketch the graph of $F(x)$ and $F'(x)$.

(c) Find the Fourier series of $F(x)$

3. (20%)

Evaluate $\iint x\,dydz+y\,dzdx+z\,dxdy$ over the surface $z=1-x^2-y^2$ for $x^2+y^2\le1$, oriented by the upper normal.

4. (20%)

Please solve the initial value problem

$$y''' - 2y'' - y' + 2y = 2x^2 - 6x + 4$$

$$y(0) = 5 \quad y'(0) = -5 \quad y''(0) = 1$$

5. (20%)

A system of second-order differential equation is given by

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} y_1'' \\ y_2'' \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}.$$

(a) Assume that the solution is of the form $\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix} e^{i\omega t}$ if $f(t) = 0$. Find the solution $\begin{Bmatrix} y_1(t) \\ y_2(t) \end{Bmatrix}$ satisfying the initial conditions

$$\begin{Bmatrix} y_1(0) \\ y_2(0) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 3 \end{Bmatrix}, \quad \begin{Bmatrix} y_1'(0) \\ y_2'(0) \end{Bmatrix} = \begin{Bmatrix} -\sqrt{10} \\ 0 \end{Bmatrix}.$$

(b) If $f(t) = \sin(\omega_o t)$, determine the range of ω_o such that the particular solution $\begin{Bmatrix} y_{1p}(t) \\ y_{2p}(t) \end{Bmatrix}$ are in the same phase, *i.e.*, both $y_{1p}(t)$ and $y_{2p}(t)$ have the same sign.