There are 100 points in total.

1. Solve the ODE.

$$x^2 y'' - 4xy' + 6y = -7x^4 \sin x \quad \text{(15 points)}$$

2. Use the power series method to show that one of the solutions of the following Bessel's equation is $y = \sin x / \sqrt{x}$

$$x^2 y'' + xy' + (x^2 - \frac{1}{4})y = 0 \quad \text{(20 points)}$$

(Note: $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} \cdots$)

3. Evaluate the flux integral $\displaystyle\iint_S \vec{F} \cdot \vec{n} \, dA$ where $\vec{F} = x^2 \vec{i} + e^y \vec{j} + \vec{k}$ and

S: 2x+4y+z=1, $x \geq 0, y \geq 0, z \geq 0$. (15 points)

4. (a) Evaluate the inverse Fourier Transform $F^{-1}\left\{\dfrac{2}{\omega^2 + 1}\right\}$ (15 points)

(b) Solve $EIy''' + ky = w(x)$ using Fourier Transform by expressing the solution in terms of integral form. Note E, I and k are physical constants. (15 points) Hint: You may need the following formula

$$F\{e^{-a|x|/\sqrt{2}} \sin(\frac{a}{\sqrt{2}}|x| + \frac{\pi}{4})\} = \frac{2a^3}{\omega^4 + a^4} \quad (a>0) \text{ in solving the above ODE.}$$

5. Solve the following partial differential equation,

$$\alpha^2 v_{xx} = v_t + hv \quad (0<x<L,\ 0<t<\infty)$$
$$v(0,t) = v(L,t) = 100 \quad v(x,0) = f(x)$$

by separation of variables, leaving expansion coefficients in integral form. (20 points)