

國立清華大學 100 學年度碩士班入學考試試題

系所班組別：生命科學院丙組

考試科目（代碼）：微積分(0601)

共 2 頁，第 1 頁 \*請在【答案卷】作答

15% (1) Evaluate the integral

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \sin(x^2 + y^2) dx dy$$

15% (2) Evaluate the integral

$$\int \frac{1}{x^3-1} dx$$

10% (3) Let  $\Phi = \Phi(u)$  be differentiable. Let  $z = z(x, y)$  be defined as the solution of

$$ax + by + cz = \Phi(x^2 + y^2 + z^2).$$

Show that  $z = z(x, y)$  satisfies

$$(cy - bz) \frac{\partial z}{\partial x} + (az - cx) \frac{\partial z}{\partial y} = bx - ay.$$

10% (4) Evaluate the limit

$$\lim_{s \rightarrow \infty} \left( \frac{a^s + b^s}{2} \right)^{1/s} \quad \text{where } 0 < a < b$$

15% (5) Sketch the graph of  $y = \frac{x+1}{x^2+1}$ . Find the intervals where  $y$  is increasing,

$y$  is decreasing,  $y$  is concave downward,  $y$  is concave upward, the local minimum, local maximum and inflection points.

15% (6) Expand  $f(x) = \tan^{-1} x$  into Taylor series at  $x = 0$ , then integrate the series term by term to find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$ .

10% (7) Given  $n$  points  $M_i(x_i, y_i, z_i)$ ,  $i = 1, 2, \dots, n$ . Find a point  $P$  on the

sphere  $x^2 + y^2 + z^2 = 1$  such that  $\sum_{i=1}^n (\overline{PM})^2$  is minimum.

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共 2 頁，第 2 頁 \*請在【答案卷】作答

10% (8) Let  $n_1 = 8, n_2 = 6, n_3 = 3, P_{AA} = \theta^2, P_{Aa} = 2\theta(1 - \theta),$   
 $P_{aa} = (1 - \theta)^2, \quad 0 < \theta < 1. L(\theta) = P_{AA}^{n_1} P_{Aa}^{n_2} P_{aa}^{n_3}.$

- (i) Show that if  $L(\theta)$  is maximal for  $\theta = \hat{\theta}$  if and only if  $\ln L(\theta)$  is maximal for  $\theta = \hat{\theta}$ .
- (ii) Find the value  $\hat{\theta}$  that maximizes  $L(\theta)$ .