# 國立臺北科技大學105學年度碩士班招生考試

## 系所組別：3110、3120、3150
## 土木工程系土木與防災碩士班甲、乙、戊組

## 第三節　工程數學　試題



**注意事項：**

1. 本試題共5題，共100分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 計算條件若有不足，請自行作合理假設。

一. Solve the following differential equations: (3 x 10%，共30%)

1. $(2x+y-2)dx-(4x+2y+1)dy=0$
2. $e^{y}(1+x^{2})dy-2xdx=0$
3. $x^{3}\frac{d^{3}y}{dx^{3}}+5x^{2}\frac{d^{2}y}{dx^{2}}+7x\frac{dy}{dx}+8y=0$

二. Given that $y_1(x)=\cos x$ and $y_2(x)=\sin x$ form a fundamental set of solutions for the corresponding homogeneous differential equation of the following equation, find its general solution: (20%)

$$y''+y=4x+10\cos x,\quad y(\pi)=0,\ y'(\pi)=4$$

三. Given that $A=\begin{bmatrix}0 & 2 & 3\\ 2 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}$: (5 x 4%，共20%)

1. Find the determinant, $|A|$
2. Find the inverse, $A^{-1}$;
3. Find the eigenvalues and the corresponding eigenvectors of $A$;
4. Find a matrix $P$ such that $D=P^{-1}AP$, where $D$ is a diagonal matrix;
5. Find the eigenvalues of the matrix $A^3+3A^2+6A$。

四. Solve the following system of differential equations by Laplace Transform: (20%)

$$y_1'' = -2y_1 + 2(y_2 - y_1)$$
$$y_2'' = -2(y_2 - y_1) - 2y_2$$

$$y_1(0) = y_2(0) = 1, \qquad y_1'(0) = \sqrt{6}, \qquad y_2'(0) = -\sqrt{6}$$

五. Expand the following function in a Fourier Series: (10%)

$$f(x) = x \quad \text{for} \quad -\pi \leq x \leq \pi$$