國立臺北科技大學 105 學年度碩士班招生考試

系所組別：1521、1522、1523 自動化科技研究所乙組

第二節　工程數學　試題

> **注意事項：**
> 1. 本試題共六題，共 100 分。
> 2. 請標明大題、子題編號作答，不必抄題。
> 3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (20%) Consider the following system of linear equations:

$$x + y + z - w = -1$$
$$y + z + w = 0$$
$$x + y + z \quad\quad = 0$$

   (a) Give the augmented matrix of the system. (5%)

   (b) Put the augmented matrix in reduced row echelon form. (5%)

   (c) Give all solutions to the system of equations. (10%)

2. (20%) Let $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$

   (a) Find a least-squares solution of $Ax = b$. (10%)

   (b) Find the least squares error associated with the solution. (10%)

3. (20%) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, Find A^{-1}

4. (20%) Consider the elementary row operation where row 2 is replaced by row 2 minus twice row 4. Let E be the elementary matrix corresponding to this operation. In other words we may apply this operation to the 4×4 matrix A by computing EA.

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & c & 0 & d \end{bmatrix}$$

Find the values of a, b, c, and d.

5. (10%) Find the eigenvalues of A^9 for

$$A = \begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & 1/2 & 3 & 8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

6. (10%) Consider the system

$$2x_1 - 3x_2 + 5x_3 = 0$$
$$-x_1 + 7x_2 - x_3 = 0$$
$$4x_1 - 11x_2 + kx_3 = 0$$

For what value of k will the system have nontrivial solutions?