國立清華大學 100 學年度碩士班入學考試試題系所班組別：聯合招生(工科丙組、先進光源工科組)

考試科目（代碼）：電磁學(9803)

1. A long coaxial cable has an inner conducting *shell* (infinitely thin) of diameter a (= 1 cm) and an outer conducting *shell* of diameter b (= 2 cm), as shown in Fig. 1. The region between the conductors is filled with a medium of *relative* permittivity $\varepsilon_r = 2$, and relative permeability $\mu_r = 5$. A charge (per unit length) of $\lambda_1 = 1$ C/m is placed on the inner conducting shell, while $\lambda_2 = -1$ C/m on the outer conducting shell. Charges on both conducting shells are moving with a velocity, $\vec{v}_1 = 1.0\,\hat{\phi} + 1.0\,\hat{z}$ m/s (inner), and $\vec{v}_2 = -1.0\,\hat{\phi} + 1.0\,\hat{z}$ m/s (outer) (expressed in cylindrical coordinates, z-axis being the center axis of the cable, as shown in Fig. 1). Find (20 %)

(a) *electric field* ($\vec{E}$) everywhere.

(b) electrostatic *potential energy* (per unit length) stored in the cable.

(c) *polarization* and *bound* charges (densities) everywhere.

(d) *capacitance* (per unit length) of the cable.

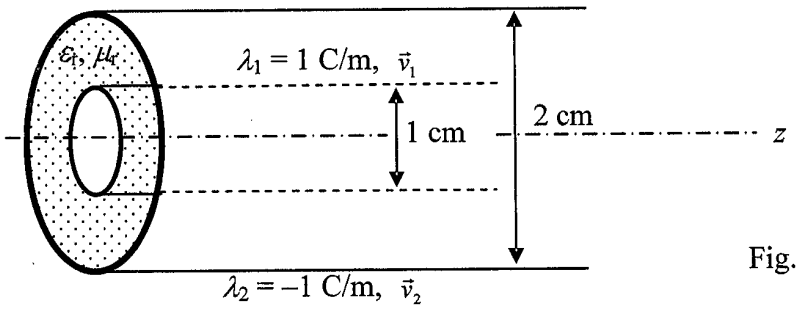

Fig. 1

2. For the cable in problem 1 (Fig. 1), find (20%)

(a) *magnetic field* ($\vec{B}$) everywhere,

(b) *bound currents* everywhere,

(c) *magnetic energy* (per unit length) stored in the cable,

(d) *inductance* (per unit length) of the cable.

3. Consider a hollow infinite long rectangular pipe formed by perfect conductors of width a and height b, as illustrated in Fig. 2. The four sides of the pipe are held at different electric potentials, as indicated in Fig. 2 and the insider of the pipe is filled with a medium of *relative* permittivity $\varepsilon_r = 1$, and relative permeability $\mu_r = 1$. Find the electric potential distribution inside the pipe. (20 %)

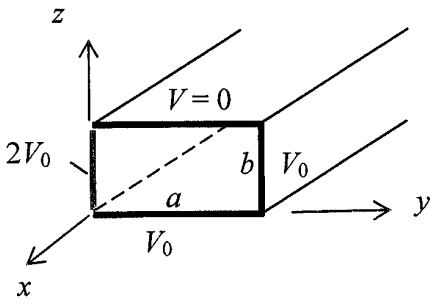

Fig. 2

4. Now consider the electromagnetic waves propagating in the rectangular conducting pipe in Fig. 2 (four conducting walls are now connected together and can be considered a "single" conductor). (20 %)
   (a) Can the transverse electromagnetic wave (TEM) propagate along the waveguide? Explain your answer.
   (b) For $a = 6$ cm, $b = 3$ cm, what is the *waveguide mode* with the *lowest* cutoff frequency? (mode type (TE or TM), index number and corresponding cutoff frequency)
   (c) Continue from (b), for a wave propagating along this waveguide under the fundamental mode and at a wave frequency $f = 5$ GHz, find the corresponding *guide wavelength*, *phase* velocity and the *group* velocity.
   (d) One of the velocities (phase or group velocity) in (c) is higher than the speed of light in vacuum. Does this violate the special relativity? Explain your answer.

5. Consider a monochromatic plane wave propagating in a "*good conductor*" of conductivity. (20 %)

(a) What is the *criterion* (in terms of conductivity, permittivity of the conductor and the wave angular frequency) for a "good" conductor? Explain your answer. Write down the corresponding Maxwell's equations for fields in a good conductor.

(b) Find the plane wave solution. (in complex form)

(c) What is the "skin depth"? Show that the skin depth in a good conductor is $\lambda/2\pi$, where λ is the wavelength in the conductor.

(d) If the intensity at $z = 0$ of a plane wave of frequency 1 GHz, propagating in silver ($\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 6\times10^6\ (\Omega\cdot\text{m})^{-1}$, propagating along the $+z$-axis) is 1 W/m^2, find the wave intensity at $z = 2$ μm.