

# 國立臺灣師範大學 105 學年度碩士班招生考試試題

科目：工程數學

適用系所：機電工程學系-光機電系統組

注意：1.本試題共 3 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. Consider a first-order system described by

$$\frac{dy}{dt} + 2y = x(t) \text{ subject to } y(0^-) = 1,$$

in which  $y(t)$  and  $x(t)$  denote the system's output and input, respectively. Find  $x(t)$ , which can yield the output  $y(t) = 1 + \exp(-2t)$  for  $t > 0$ . (10 分)

2. A 1-kilogram mass is attached to a spring whose constant is 8 N/m, and the whole system is submerged in a liquid that gives a damping force equal to 6 times the instantaneous velocity of the mass. (10 分)

(a) Find the equation of motion if the mass is initially released from a point 1 meter below the equilibrium position with an upward velocity of 6 m/s. (5 分)

分)

(b) Find the time at which the mass attains its extreme displacement above the equilibrium position. (5 分)

3. Consider a parallel RLC circuit with a voltage source  $v_s$  as shown in Figure 1. (15 分)

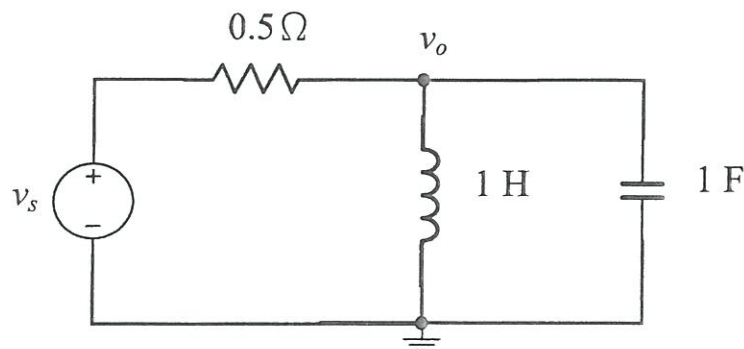


Figure 1. Network for Problem 3

(a) Find the differential equation for the output voltage  $v_o$ . (5 分)

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(b) Find the transfer function from  $v_s$  to  $v_o$ . (5 分)

(c) The network is initially at rest, that is, the total energy stored in the capacitor and the inductor is zero at  $t = 0$ . If the input voltage  $v_s(t) = t^2$  V for  $t \geq 0$ , determine  $v_o$  for  $t \geq 0$ . (5 分)

4. Consider a robot arm, moving in a plane and consisting of two rigid links pinned together at a joint that has a vertical axis. The base joint of the first link also rotates about a vertical axis. Hence, the kinematic structure has two revolute joints, and the joint angles can be denoted as  $\theta_1$  and  $\theta_2$ . Here, the relation between the end-effector position  $(x, y)$  and the joint angles is given by

$$(x, y) = (\cos \theta_1 + \cos(\theta_1 + \theta_2), \sin \theta_1 + \sin(\theta_1 + \theta_2)). \quad (20 \text{ 分})$$

(a) Find a matrix  $\mathbf{J}(\theta_1, \theta_2)$  that satisfies the relation. (5 分)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J}(\theta_1, \theta_2) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}.$$

(b) Find the eigenvalues and eigenvectors of  $\mathbf{J}(0,0)$ . (5 分)

(c) Let  $e^{\mathbf{J}(0,0)} = \alpha_0 \mathbf{I} + \alpha_1 \mathbf{J}(0,0)$ . Determine  $\alpha_0$  and  $\alpha_1$ . (5 分)

(d) Find a nonsingular matrix  $\mathbf{P}$  such that  $\mathbf{P}^{-1} \mathbf{J}(0,0) \mathbf{P}$  is a diagonal matrix. (5 分)

5. Consider the double integral  $\int_0^2 \int_{y^2}^4 \cos \sqrt{x^3} dx dy$ . (15 分)

(a) Sketch the region of integration for the given iterated integral. (5 分)

(b) Evaluate the given iterated integral by changing or reversing the order of integration. (10 分)

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6. The temperature  $T$  at a point  $(x, y, z)$  in space is inversely proportional to the square of the distance from  $(x, y, z)$  to the origin. It is known that  $T(1, 1, 1) = 100$ .

(15 分)

(a) Find the rate of change of  $T$  at  $(2, 1, 1)$  in the direction of  $(3, 1, 2)$ . (5 分)

(b) In which direction from  $(2, 1, 1)$  does the temperature  $T$  increase most rapidly? (5 分)

(c) At  $(2, 1, 1)$  what is the maximum rate of change of  $T$ ? (5 分)

7. Consider  $y'' + \lambda y = 0$  subject to  $y'(0) = 0$  and  $y'(1) = 0$ . (15 分)

(a) Show that the eigenfunctions are

$$\{1, \cos \pi x, \cos 2\pi x, \dots\}. \quad (5 \text{ 分})$$

(b) Is the set orthogonal on  $[0, 1]$ ? Give a detailed explanation. (5 分)

(c) Find the square norm of each eigenfunction. (5 分)