1. Find the following limits respectively. If it does not exist, explain why.

   (a) $\lim_{x \to 1}(\dfrac{1}{\ln x} - \dfrac{1}{x-1})$ (5 pts)

   (b) $\lim_{x \to 0}\dfrac{2x^2}{1 - \cos 3x}$ (5 pts)

   (c) $\lim_{x \to \infty}(\dfrac{2}{\pi}\tan^{-1}x)^x$ (5 pts)

2. Evaluate $\displaystyle\int_0^1 x^{15}\sqrt{1+3x^8}\,dx$ (10 pts)

3. Evaluate $\displaystyle\int e^x \cos x\,dx$ (10 pts)

4. What is the area of the largest rectangle in the first quadrant with two sides on the axes and one vertex on the curve $y = e^{-2x}$. (15 pts)

5. In 3D space (x,y,z), the plane P: x+y+z=1 cuts the cylinder C:x^2+y^2=1 in an ellipse E. Using Lagrange Multipliers to find the points on the ellipse E that lie closet to and farthest from the origin. (15 pts)

6. If resistors of R_1, R_2, and R_3 ohms are connected in parallel to make an R-ohm resistor (see the figure), the value of R can be found from the equation:

   $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

   Please find the value of $\dfrac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 20$ and $R_3 = 60$ ohms. (10 pts)
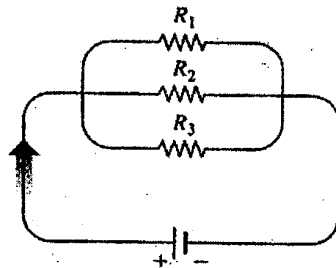
7. Write the expression of the tangent plane and calculate the normal vector of the

surface $f(x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at a point $(1, 2, 4)$. (10 pts)

8. Find the local extreme values of the following function:

$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4 = 0$. (15 pts)