# 國立清華大學 100 學年度碩士班入學考試試題

系所班組別：工業工程與工程管理學系(甲組)

考試科目（代碼）：作業研究（1602）

　*請在【答案卷】作答

注意事項：(1) 不得使用計算器。
(2) 請依題號順序作答。
(3) 答案必須寫在**答案卷**上，並須依每一題規定的方式作答。
(4) 未依規定方式作答，酌量扣分。

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1. Consider the following linear program.

$$\begin{aligned} \text{Minimize} \quad & z = c_1x_1 + c_2x_2 \\ \text{subject to} \quad & a_{11}x_1 + a_{12}x_2 \geq b_1 \\ & a_{21}x_1 + a_{22}x_2 = b_2, \; x_1, x_2 \geq 0. \end{aligned}$$

Suppose we compute the optimal tableau by the Big M method, where we let $x_3$ be the slack variable of constraint 1, and $x_4$ and $x_5$ the artificial variables of constraints 1 and 2 respectively.

| Basic Variable | Coefficients of: | | | | | | RHS |
|---|---|---|---|---|---|---|---|
| | $z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
| $z$ | 1 | 0 | 0 | $-\frac{9}{7}$ | $\frac{9}{7}-M$ | $\frac{10}{7}-M$ | 18 |
| $x_1$ | 0 | 1 | 0 | $-\frac{3}{7}$ | $\frac{3}{7}$ | $\frac{1}{7}$ | 3 |
| $x_2$ | 0 | 0 | 1 | $\frac{1}{7}$ | $-\frac{1}{7}$ | $\frac{2}{7}$ | 2 |

Please determine $b_1$, $b_2$, $a_{11}$, $a_{12}$, $a_{21}$, $a_{22}$, $c_1$ and $c_2$.
必須在**答案卷**畫出以下表格並在表格內填寫答案。

| $(b_1, b_2)$ | $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ | $(c_1, c_2)$ |
|---|---|---|
| | | |

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2. Consider the following parametric linear programming problem with parametric change on the objective function.

$$\begin{aligned} \text{Maximize} \quad & z = (5+3t)x_1 + (6+5t)x_2 + (8-10t)x_3 \\ \text{subject to} \quad & x_1 + 2x_2 + x_3 \le 19 \\ & x_1 + x_2 + 3x_3 \le 27,\ x_1, x_2, x_3 \ge 0. \end{aligned}$$

(2-1) Determine the optimal basic feasible solution for $t = 0$. (5%)

(2-2) Determine the range $[t_1, t_2]$ such that the above basic feasible solution (2-1) remains optimal over the range $t_1 \le t \le t_2$. Also find the objective value $f(t)$ as a function of $t$. (10%)

必須在答案卷畫出以下表格並在表格內填寫答案。

| optimal solution $(z, x_1, x_2, x_3)$ for $t = 0$ | $[t_1, t_2]$ | $f(t)$ |
|---|---|---|
| | | |

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3. A distributor wants to build several distribution centers to interconnect each of its retailers by a connected network of shortest length, where the length is the sum of the lengths of all links between them. This is the so-called *Steiner Tree* problem (see Figure 1). Given a set R of retailers and a set S of possible locations for distribution centers (*Steiner points*), the problem is to select a subset of S to interconnect all retailers in R with shortest length under the distance function $d$: E $\rightarrow Z^+$, where E is the set of links and $Z^+$ is the set of positive integers.

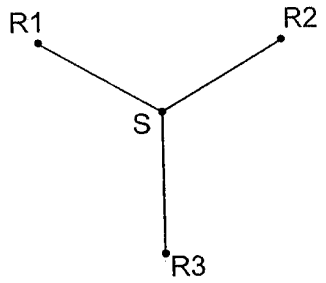


Figure 1. An example of a Steiner Tree with three retailers and a Steiner point.

(3-1) Try to formulate a mathematical programming model for this minimization problem. (5%)

(3-2) Write down the corresponding dual program of the above model. (5%)

(3-3) Formulate a mathematical programming model for the *Minimum Spanning Tree* (MST) problem which includes all nodes in R. (5%)

(3-4) Distinguish the difference between these two minimization problems. More specifically, prove or disprove if a feasible solution of MST is a feasible solution of Steiner Tree and vice versa. (5%)

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4. A certain shop has two identical machines that are operated continuously except when they are broken down. Because they break down fairly frequently, the top-priority assignment for a full-time maintenance person is to repair them whenever needed. The time required to repair a machine has an exponential distribution with a mean of 1/2 day. Once the repair a machine is completed, the time until the next breakdown of that machine has an exponential distribution with a mean of 1 day. These distributions are independent.

(4-1) Develop the rate diagram for this Markov chain. (5%)

(4-2) Construct the steady-state equations. (5%)

(4-3) Solve these equations for the steady-state probabilities. (5%)

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5. You are given an M/M/1 queueing system in which the expected waiting time and expected number in the system are 120 minutes and 8 customers, respectively.
   (5-1) What is the mean arrival rate of the queueing system? (5%)
   (5-2) What is the mean service rate of the queueing system? (5%)
   (5-3) Determine the probability that a customer's waiting time in the system exceeds 20 minutes. (10%)

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6. Machinists who work at a tool-and-die plant must check out tools from a tool center. An average of ten machinists per hour arrive seeking parts. At present, the tool center is staffed by a clerk who takes an average of 5 minutes to handle each request for tools. Since machinists are paid $10 per hour, each hour that a machinist spends at the tool center costs the company $10. The company is deciding whether or not it is worthwhile to hire (at $4 per hour) a helper for the clerk. If the helper is hired, the clerk will take an average of only 4 minutes to process requests for tools. Assume that service and inter-arrival times are exponential. Based on the cost analysis, should the helper be hired?