東海大學105學年度碩士班考試入學試題

考試科目： 微積分A　　　　科目代碼： 24011
應考系組： 應數系
考試日期：105年03月06日第3節　　使用計算機：不可　　共 2 頁(第 1 頁)

(一) 填充題($14 \times 5\%$)　　**請於答案卷作答，違者不予計分**

1. Find the limit (極限) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x =$ ____________________.

2. Find the increasing and decreasing intervals (上升和下降區間) and the concave upward and downward intervals (上凹和下凹區間) of the function $f(x) = \dfrac{x}{1+x^2}$.
   上升區間____________________, 下降區間____________________.
   上凹區間____________________, 下凹區間____________________.

3. Find the area (面積) A below the curve $y = \sin^{-1} x$ for $0 \le x \le 1/2$.
   $A =$ ____________________.

4. Find an equation of the tangent line (切線方程式) to the following curve at the point $(2,-3)$.
   $$x^2+y^2-2x+4y=-3.$$
   切線方程式爲____________________.

5. Use the linear approximation (線性近似) of $f(x,y)=\tan[\pi(x^2-y^3)]$ at $(3,2)$ to approximate $f(2.9,2.1)\approx$ ____________________.

6. Find the directional derivative (方向導數) of $f(x,y)=\ln(xy+x^2)$ at the point $(2,2)$ in the direction of $\mathrm{v}=(1,2)$.
   $D_{\mathrm{v}}f(2,2)=$ ____________________.

7. Find the local maxima, minima, and saddle points (極大,極小,和馬鞍點) of $f(x,y)=3x^2y+y^3-3x^2-3y^2+2$.
   極大____________________, 極小____________________,
   馬鞍點____________________.

8. Evaluate by reversing the order of integration (交換積分順序) the following integral $\int_0^1\int_{\sqrt{y}}^1 e^{-x^3}\,dx\,dy =$ ____________________.

9. Use the transformations (轉換) $u=x+2y$ and $v=2x-4y$ to evaluate
   $$\iint_R \sqrt{\frac{x+2y}{2x-4y}}\,dx\,dy = \_\_\_\_\_\_\_\_\_\_,$$
   where $R=\{(x,y)\mid 1\le x+2y\le 4,\ 1\le 2x-4y\le 16\}$.

(二) 計算題 (3×10%)

10. i. Find a function f such that $\overrightarrow{F} = \nabla f$, where

$$\overrightarrow{F} = \Big(y^2 \cos z,\ 2xy \cos z,\ -xy^2 \sin z - 3 \Big).$$

ii. Use part (a) to evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$, where $C : \overrightarrow{r}(t) = (\cos t,\ \sin t,\ t)$ and $0 \le t \le \pi/4$.

11. Use Green's Theorem to evaluate the line integral

$$\int_C xe^{-2x}\, dx + (x^4 + 2x^2y^2)\, dy$$

where C is the boundary of the region in the first quadrant (第一象限) between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, and C is oriented positively.

12. Use the Divergence Theorem (Gauss Theorem) to evaluate the flux $\iint_S \overrightarrow{F} \cdot d\overrightarrow{S}$, where

$$\overrightarrow{F} = \Big(z \tan^{-1}(y^2),\ z^3 \ln(x^2 + 1),\ z \Big)$$

and S is boundary (邊界) of the solid enclosed by the paraboloid (抛物面) $z = 1 - x^2 - y^2$ and the xy-plane.