招 生 學 年 度	105	招 生 類 別	碩士班
系 所 班 別	應用數學系碩士班		
科 目 名 稱	線性代數		
注 意 事 項	本考科禁止使用掌上型計算機		

1. (15%) Show that $\{(1,1,0),(1,1,1),(0,1,-1)\}$ is linearly independent.

2. (10%) If $W = \{(a_1, a_2, a_3, a_4) \in \mathbb{R}^4 : a_1 = a_2 = a_3\}$ is a subspace of $\mathbb{R}^4$, find its dimension.

3. Let $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$.

   (a) (5%) Calculate $\det A$.

   (b) (10%) Determine whether A is invertible, and find A^{-1} if it exists.

4. (15%) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be an 2×2 real matrix, the trace of A is defined by

$$\mathbf{tr}(A) = a + d.$$

   Calculate $\mathbf{tr}(AB)$ if $A = \begin{pmatrix} 0 & 1 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$.

5. (15%) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformations defined by

$$T(a,b) = (3a + b, a - 2b),$$

   let $\gamma = \{(1,1),(0,1)\}$ and $\beta = \{(1,0),(0,1)\}$ be ordered bases for $\mathbb{R}^2$. Let $[T]_\gamma^\beta$ be the matrix representation of T in the ordered bases γ and β. Find the matrix $[T]_\gamma^\beta$.

6. (15%) Solve the given system of linear equations.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 2, \\ x_1 \quad\quad + x_3 &= 3, \\ x_1 + x_2 - x_3 &= 1, \end{aligned}$$

7. (15%) For what value of a is

$$\det \begin{pmatrix} 1 & 5 & a & 0 \\ 2 & 6 & 10 & 0 \\ 3 & 7 & 0 & a \\ 4 & 8 & 0 & a \end{pmatrix} = 0.$$