For Questions 1 to 5, show your calculations **in detail** on the answer sheet.

1. (10%) Find $\frac{dF(x)}{dx}$ if $F(x) = \sqrt{x^2+1}$.

2. (10%) If $f(x,y) = x^3 + x^2y^3 - 2y^2$, find $f_x(2,1)$ and $f_y(2,1)$.

3. (10%) Find $\frac{d}{dx}[\ln|\cos(x)|]$.

4. (10%) If $w = x^3y^4$ and $x = t^2 - 2$, $y = 5t - 3$, find $\frac{dw}{dt}$ at $t = 1$.

5. (10%) Find $\int \ln x \, dx$.

For Questions 6 to 15, select a correct answer for each question and mark the letter (A), (B), (C), or (D) on your answer card.

6. (5%) What is the value of $x + 2^2x^2 + \cdots + n^2x^n$ if $x = 3$ and $n = 12$?
   (A) 106022478
   (B) 108070187
   (C) 108080187
   (D) None of the above

7. (5%) For the summation equation in Question 6, what is its value if $x = 0.5$ and $n \to \infty$?
   (A) 5
   (B) 6
   (C) 7
   (D) None of the above

8. (5%) Determine whether the following statement is true or false.

   "Suppose a_n is a series. If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = c$, then $\lim_{n\to\infty} \sqrt[n]{|a_n|} = c$."

   (A) False
   (B) True
   (C) Uncertain (need more information)
   (D) None of the above

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9. (5%) The Gaussian Quadrature method is a numerical way to generate the approximation of the definite integral of any function $f(x)$ as follows.

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i).$$

Both the abscissas, x_i, and the weighting coefficients, w_i, are free parameters and they can be solved based on the following $2n$ equations

$$\int_{-1}^{1} x^l\, dx = \sum_{i=1}^{n} w_i x_i^l \text{ for } l = 0,1,\ldots,2n-1.$$

If $n = 2$, what is the value of $w_1 + w_2 + x_1 + x_2$?

(A) 0

(B) 1

(C) 2

(D) None of the above

10. (5%) Given the solutions of w_1, w_2, x_1, and x_2 in Question 9, use the Gaussian Quadrature method (with $n = 2$) to approximate $\int_{-1}^{1} x\ln(x^2)dx$. What is the absolute value of the approximation error?

(A) $\ln(1/3)$

(B) $\sqrt{1/3}$

(C) 0

(D) None of the above

11. (5%) Suppose that a random variable S follows a lognormal distribution and its probability density function is

$$\frac{1}{S\sigma\sqrt{2\pi}} e^{-\frac{(\ln S-\mu)^2}{2\sigma^2}},$$

where μ and σ represent the mean and standard deviation of the lognormal distribution, respectively. Define $N(d)$ as the cumulative probability of the standard normal distribution from $-\infty$ to a constant d. What is the expected value of S conditional on $S \geq K$, where K is a nonnegative constant?

(A) $e^{\mu+0.5\sigma^2} N\left(\frac{\mu+\sigma^2-\ln K}{\sigma}\right)$

(B) $e^{\mu+\sigma^2} N\left(\frac{\mu+0.5\sigma^2-\ln K}{\sigma}\right)$

(C) $e^{\mu+0.5\sigma^2} N\left(\frac{\mu+0.5\sigma^2-\ln K}{\sigma}\right)$

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(D) None of the above

12. (5%) A company grows in value by 10% each year, and also gains 20% of a growing market estimated at $100e^{0.1t}$ million dollars, where t is the number of years that the company has been in business. Therefore, the value $y(t)$ of the company (in millions of dollars) satisfies

$$y' = 0.1y + 20e^{0.1t} \text{ and } y(0) = 1 \text{ (million dollars).}$$

What is the value of this company after 5 years?

(A) 173 million dollars
(B) 167 million dollars
(C) 180 million dollars
(D) None of the above

13. (5%) Let $f(x,y) = 2x^2 + y^2 + 2xy + 4x + 2y + 7$. Find the minimum value of the function $f(x,y)$ subject to the constraint $4x^2 + 4xy = 1$.

(A) 4
(B) 5
(C) 6
(D) None of the above

14. (5%) Evaluate the following double integral for a specified region A.

$$\iint_A e^{y^3} dA,$$

where A is the region bounded by $y = \sqrt{x}$, $y = 1$, and $x = 0$.

(A) $\frac{1}{2}(e-1)$

(B) $\frac{1}{3}(e-2)$

(C) 1
(D) None of the above

15. (5%) What is the value of $\int_1^\infty \frac{\ln x}{x^2} dx$?

(A) 0
(B) ∞
(C) 1
(D) None of the above

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