- 本試題共 6 大題, 合計 100 分。
- 請依題號依序作答。
- 請詳述理由或計算推導過程, 否則不予計分。

1. (20%) Given a random sample $\{X_i\}_{i=1}^n \sim^{i.i.d.} F_X(x)$, where $F_X(x)$ is the distribution function. Define an empirical distribution function as

$$\hat{F}_n(x) = \frac{\sum_{i=1}^n I_{\{X_i \leq x\}}}{n},$$

where $I_{\{X_i \leq x\}}$ is an indicator function.

   (a) (5%) Interpret the meaning of an empirical distribution function in plain words.

   (b) (5%) Is $\hat{F}_n(x)$ an unbiased estimator of $F_X(x)$?

   (c) (5%) Is $\hat{F}_n(x)$ a consistent estimator of $F_X(x)$?

   (d) (5%) Find an asymptotic distribution of $\hat{F}_n(x)$.

2. (20%) In order to investigate the impact of mortgage rates on housing prices in Taiwan, an economist decides to regress P_i, the average housing price in city i, on a constant and R_i, the average mortgage rate in city i. Suppose that we have a random sample $\{P_i, R_i\}_{i=1}^n$

   (a) (5%) Write down the linear regression model the economist wants to estimate.

   (b) (5%) When can the economist conclude that there is a causal relationship between mortgage rates and housing prices?

   (c) (5%) Based on the assumption in (b), find the conditional expectation $E(P|R)$.

   (d) (5%) Find the estimators of the regression coefficients using the method of moments.

3. (10%) Suppose that X is a random variable.

   (a) (5%) Use 1st-order Taylor approximation to show

$$\log E(X) \approx E(\log X)$$

   (b) (5%) Suppose that X is a log-normal random variable. Show that

$$\log E(X) = E(\log X) + \frac{1}{2} Var(\log X)$$

4. (20%) **True** or **False**, and Why? Evaluate the following statements with **brief** explanations.

   (a) (5%) The most important task in obtaining a true causality is to maxmize $\bar{R}^2$.

   (b) (5%) If a regression suffers from perfect multicolinearity, the OLS estimator will be very imprecise.

   (c) (5%) The difference between R^2 and $\bar{R}^2$ is that homoskedasticity of the error terms is assumed in computing $\bar{R}^2$.

   (d) (5%) If one of the Gauss-Markov conditions that $\text{Var}(u_i|X_i) = \sigma_u^2$ is replaced with $\text{Var}(u_i|X_i) = \theta_0 + \theta_1 X_i + \theta_2 X_i^2$, $\theta_0 > 0$, $\theta_1 > 0$ and $\theta_2 > 0$, then the OLS estimator of β_1 is not consistent.

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5. (10%) Let $Y_i \sim^{i.i.d.} (\mu_Y, \sigma_Y^2)$ and $\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$. Answer the following questions.

(a) (5%) $\overline{Y}$ is an unbiased estimator of μ_Y. Is $\overline{Y}^2$ an unbiased estimator of μ_Y^2?

(b) (5%) $\overline{Y}$ is a consistent estimator of μ_Y. Is $\overline{Y}^2$ a consistent estimator of μ_Y^2?

6. (20%) Suppose that that the true model is given by equation (1)

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i, \tag{1}$$

but we mistakenly take equation (2) as our model,

$$Y_i = \beta_0^* + \beta_1^* X_{1i} + u_i^*. \tag{2}$$

(a) (4%) Calculate the OLS estimators for equation (2), i.e., $\widehat{\beta}_0^*$ and $\widehat{\beta}_1^*$.

(b) (4%) Find the relationship between $\widehat{\beta}_1^*$ and β_1.

(c) (4%) Is $\widehat{\beta}_1^*$ an unbiased estimator of β_1? Why?

(d) (4%) Is $\widehat{\beta}_1^*$ a consistent estimator of β_1? Why?

(e) (4%) From (d), under what condition(s) will $\widehat{\beta}_1^*$ be a consistent estimator of β_1?

試題隨卷繳回