1. $\boldsymbol{EFG}=\boldsymbol{H}$ given that

$$\boldsymbol{E}=\begin{bmatrix}2&0\\0&1\\0&-1\end{bmatrix},\boldsymbol{G}=\begin{bmatrix}1&-2&1\\4&3&-2\end{bmatrix},\boldsymbol{H}=\begin{bmatrix}8&6&-4\\6&-1&0\\-6&1&0\end{bmatrix}.$$

a. (10 %) Find a matrix $\boldsymbol{F}$ to satisfy this equation.
b. (10 %) Find the eigenvalues and eigenvectors of $\boldsymbol{F}$.
c. (10 %) Find a matrix $\boldsymbol{P}$ that diagonalizes $\boldsymbol{F}$.
d. (5 %) Compute $\boldsymbol{F}^{100}$.

2. $$\begin{cases}x-3y+z=0\\2x-6y+2z=0\\3x-9y+3z=0\end{cases}$$

a. (10 %) Find a basis for the solution space of this homogeneous linear system.
b. (5 %) Find the dimension of the solution space.

3. (10 %) Find a to solve

$$\begin{vmatrix}1&2&1\\0&x&3\\-3&-6&x-5\end{vmatrix}=\begin{vmatrix}a&3\\-1&1-a\end{vmatrix}$$

4. (10 %) The plane in $\boldsymbol{R}^3$ that contains the point (1, -5, 0) and is orthogonal to the line with parametric equations $x=2t$, $y=3-5t$ and $z=7$. Find the parametric equations of this plane.

5. (20 %) Find the least squares solution of the linear equation

$$\begin{bmatrix}2&0&-1\\1&-2&2\\2&-1&0\\0&1&-1\end{bmatrix}\boldsymbol{x}=\begin{bmatrix}0\\6\\0\\6\end{bmatrix}.$$

6. (10 %) Consider the basis S ={(1,0), (1,1)}={$\boldsymbol{u}_1$,$\boldsymbol{u}_2$} in $\boldsymbol{R}^2$, and let $\boldsymbol{T}: \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^2$ be the linear operator such that $\boldsymbol{T}(\boldsymbol{u}_1)=(-1,2)$ and $\boldsymbol{T}(\boldsymbol{u}_2)=(2,-3)$. Find a formula for $\boldsymbol{T}(x, y)$.