## Linear Algebra

1. $\boldsymbol{EFG} = \boldsymbol{H}$ given that

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}, G = \begin{bmatrix} 1 & -2 & 1 \\ 4 & 3 & -2 \end{bmatrix}, H = \begin{bmatrix} 8 & 6 & -4 \\ 6 & -1 & 0 \\ -6 & 1 & 0 \end{bmatrix}.$$

a. (10 %) Find a matrix $\boldsymbol{F}$ to satisfy this equation.
b. (10 %) Find the eigenvalues and eigenvectors of $\boldsymbol{F}$.
c. (10 %) Find a matrix $P$ that diagonalizes $F$.
d. (5 %) Compute $\boldsymbol{F}^{100}$.

2. $$\begin{cases} x - 3y + z = 0 \\ 2x - 6y + 2z = 0 \\ 3x - 9y + 3z = 0 \end{cases}$$

a. (10 %) Find a basis for the solution space of this homogeneous linear system.
b. (5 %) Find the dimension of the solution space.

## Probability

1. The probability that the market goes up on Monday is 0.6; given that it went up on Monday, the probability that it goes up on Tuesday is 0.3; and, finally, given that it went up on Monday and Tuesday, the probability that it goes up on Wednesday is 0.4. Find the following probabilities.
   a. (5%) The market goes up on all three days.
   b. (5%) The market goes up on Monday and Tuesday, but not on Wednesday.

2. Suppose X and Y are random variables such that
   $$\mathrm{E}\{X\}=1,\ \mathrm{Var}\{X\}=2,\ \mathrm{E}\{Y\}=3,\ \mathrm{Var}\{Y\}=4,\ \mathrm{Cov}\{X,Y\}=1$$
   Compute the following quantities:
   a. (5%) E[X + 2Y].
   b. (5%) E[XY].
   c. (5%) Var[X − 2Y + 1].

3. (5%) Consider the random variable $X$ with the following probability density function (pdf):
   $$f_X(x)=\begin{cases} p\lambda e^{-\lambda x} & , x\geq 0 \\ (1-p)\lambda e^{\lambda x} & , x<0 \end{cases}.$$
   , where $\lambda$ and $p$ are scalars with $\lambda>0$ and $p\in[0,1]$. Find the mean of $X$.

4. The following table indicates the joint probability mass function (PMF) of random variables $X$ and $Y$:

| | X=2 | X=3 |
|---|---|---|
| Y=-1 | 1/3 | 1/6 |
| Y=0 | 1/4 | 1/4 |

   a. (5%) Find the joint CDF $F_{X,Y}(x,y)$ from the indicated joint PMF.
   b. (5%) Find the marginal CDF of $X$.
   c. (5%) Find the marginal PMF of $Y$.
   d. (5%) Find the conditional probability $P[X-Y=3\,|X=2]$.