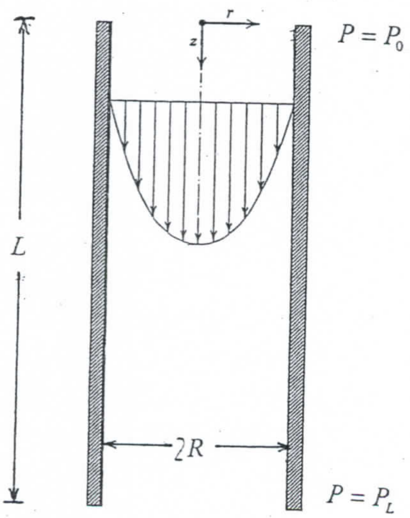
1. Consider a steady-state flow of an incompressible Newtonian fluid with constant ρ and μ through an infinitely long tube. Starting from the following equation of motion in cylindrical coordinates,

r-component
$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho g_r$$

θ-component
$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\rho g_\theta$$

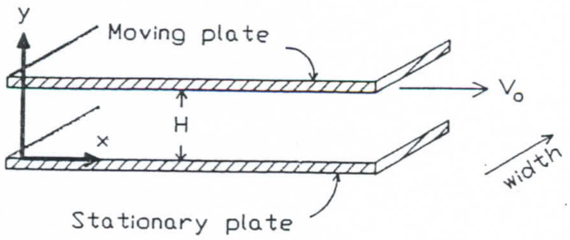
z-component
$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

(1) Prove $-\frac{\partial P}{\partial z}=\frac{P_0-P_L}{L}$ (where $P=p-\rho g z$) (10 points) total
(2) Calculate the velocity profile of the flow. (10 points) (20 points)

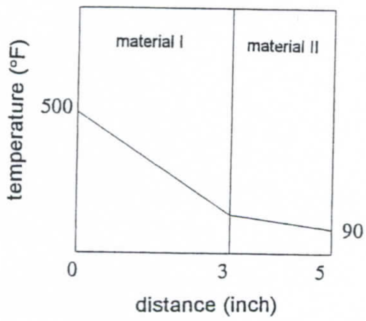


2. Consider a drag flow (i.e., no pressure variation in the direction of flow) between two horizontal parallel plates separated by a small distance H, with an incompressible Newtonian fluid in the space. One of the plates is moving with a velocity $v_x=V_o$ and the other is stationary. At steady state conditions,
(1) Prove that τ_{yx} is constant across the gap between the plates. (5 points)
(2) Calculate the velocity profile of the flow. (5 points)
(3) Find the total volumetric flow rate per unit width. (5 points)

total (15 points)



3. A wall is made up of two layers of different materials having different thermal conductivities (26.1 and 0.04 Btu/hr-ft-°F). At steady state the temperature profiles appear thus:



total (15 points)

(1) Which material has the higher thermal conductivity? Why? (5 points)
(2) Estimate the heat transfer rate through the wall. (5 points)
(3) Calculate the temperature at the interface of the two materials. (5 points)

4. Interpret the **physical meanings** of the following dimensionless groups: (a) Brinkman number (b) Prandtl number (c) Péclet number (d) Schmidt number (e) Diffusional Grashof number (total 25 points; 5 points each)

5. In studying the rate of leaching of a substance A from solid particles by a solvent B, we may postulate that the rate-controlling step is the diffusion of A from the particle surface through a stagnant liquid film thickness δ out into the main stream. The molar solubility of A in B is c_{A0}, and the concentration in the main stream is $c_{A\delta}$. Assume that the mutual diffusivity D_{AB} is constant and A is only slightly soluble in B. Neglect the curvature of the particle. (a) Write down the mass balance equation on A and, accordingly, solve (b) the concentration profile and (c) the rate of leaching. (d) Re-derive the leaching rate if the dissolved A is instantaneously removed from the solvent medium by, say, an irreversible chemical reaction. (e) Comment on how the value of δ might be estimated in practice, given the relationships obtained above. (total 25 points; 5 points each)