1. Consider a 3-arm manipulator as shown in Fig. 1. The length of each arm is 1 m.

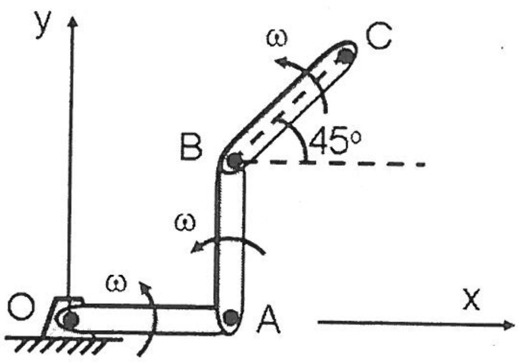

Fig. 1

(a) (15%) When each arm rotates at a constant velocity $\omega = 20$ rad/s, determine the velocity and acceleration of point C relative to the fixed point O in terms of fixed x-y coordinate system.

(b) (5%) Again each arm rotates at a constant velocity, but we do not know the value this time. An accelerometer (加速規) mounted at C indicates that the acceleration of point C relative to the fixed point O is $\mathbf{a}_c = -1500\ \mathbf{i} - 1500\ \mathbf{j}$ (m/s^2). What is the angular velocity ω? Can we determine the rotation direction of ω?

2. (10%) The right-angled bar rotates clockwise with an angular velocity ω rad/s which is decreasing at the rate of α rad/s^2 (Fig. 2). Write the vector expressions for the velocity and acceleration of point A.

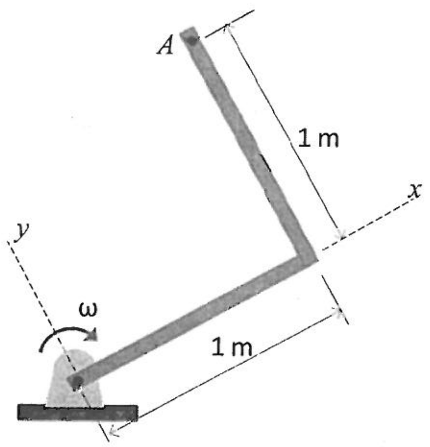

Fig. 2

3. (20%) The slender rod of mass m and length l rotates about the y-axis as the element of a right-circular cone. If the angular velocity about the y-axis is ω, determine the expression for the angular momentum of the rod with respect to the x-y-z axes for the particular position shown in Fig. 3a. *[Hint: Fig. 3b shows the mass moments of inertia of a homogeneous slender rod]*

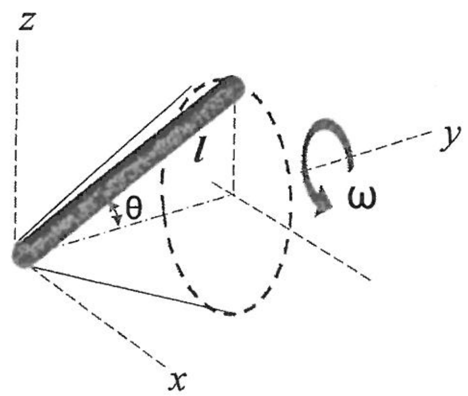

Fig. 3a

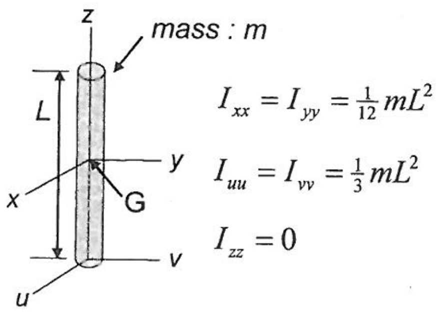

Fig. 3b

4. (20%) The slider A moves in the slot at the same time that the disk rotates about its center O with an angular speed ω positive in the clockwise sense (Fig. 4). Determine the x- and y-components of the absolute acceleration of A at the instant $\omega = 5\,\text{rad/s}$, $\dot{\omega} = -10\,\text{rad/s}^2$, $x = 50\,\text{mm}$, $\dot{x} = 150\,\text{mm/s}$, and $\ddot{x} = 500\,\text{mm/s}^2$.

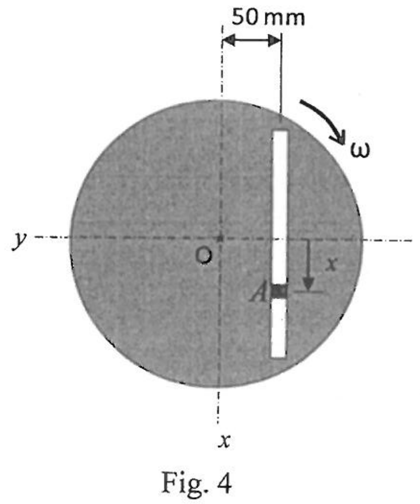

Fig. 4

5. Consider a piston inside a tube as shown in Fig. 5. The diameter of the tube is 200 mm, and the mass of the piston is 10 kg. The space to the right of the piston is a vacuum and to the left is filled with a gas. At the initial position as shown in the figure, the pressure of the gas applied on the piston is 10^5 Pa. The force F is slowly increased, moving the piston 0.5 m to the left from the initial position. The force is then removed and the piston is accelerated to the right.

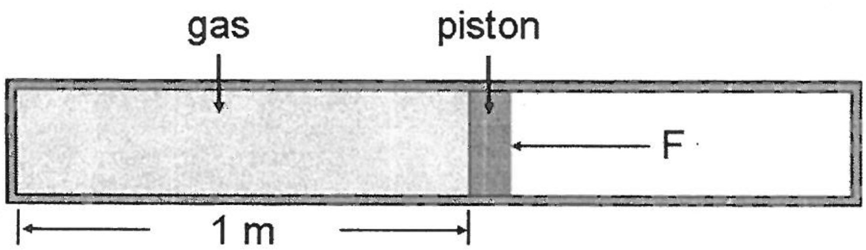

Fig. 5

(a) (10%) Assume that the friction is negligible and the pressure of the gas is related to its volume by pV = constant. (i) Write down the differential equation governing the position of the piston. (ii) Can we use the Laplace transform approach to solve the differential equation for the velocity and position of the piston as functions of time? Why?

(b) (10%) Continue with Problem (a). (i) Find the velocity of the piston as a function of piston position by using the principle of energy conservation. (ii) Can we now determine the velocity and position of the piston as functions of time? Why?

(c) (10%) Now if you assume that the pressure of the gas is related to its volume by pV = constant while it is compressed, and by $pV^{1.4}$ = constant while it is expanding, what will be the velocity of the piston as the piston returns to its initial position as shown in the above figure.