

國立中正大學 103 學年度碩士班招生考試試題

電磁晶片組

系所別：電機工程學系-計算機工程組

科目：線性代數與微分方程

電力與電能處理甲組、乙組

第 2 節

第 1 頁，共 2 頁

1. $EFG = H$ given that

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}, G = \begin{bmatrix} 1 & -2 & 1 \\ 4 & 3 & -2 \end{bmatrix}, H = \begin{bmatrix} 8 & 6 & -4 \\ 6 & -1 & 0 \\ -6 & 1 & 0 \end{bmatrix}.$$

- (10 %) Find a matrix F to satisfy this equation.
- (10 %) Find the eigenvalues and eigenvectors of F .
- (10 %) Find a matrix P that diagonalizes F .
- (5 %) Compute F^{100} .

$$2. \begin{cases} x - 3y + z = 0 \\ 2x - 6y + 2z = 0 \\ 3x - 9y + 3z = 0 \end{cases}$$

- (10 %) Find a basis for the solution space of this homogeneous linear system.
- (5 %) Find the dimension of the solution space.

Show all your work and write your answers clearly.

1. (10%) Solve the initial value problem

$$y'' - 4y' = 0, \quad y(0) = 3, \quad y'(0) = 8.$$

2. (10%) Solve the initial-value problem

$$\mathbf{X}' = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}, \quad \mathbf{X}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

3. (10%) Consider the initial value problem

$$1 + y^2 + 2(x+1)y \frac{dy}{dx} = 0, \quad y(0) = 1.$$

Verify that the differential equation is exact and solve the initial value problem.

4. (10%) Consider the initial-value problem

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4.$$

Use Euler's method to obtain a four-decimal approximation of $y(2.3)$ using $h = 0.1$ (each interval).

5. (10%) Find two linearly independent *power series solutions* of the differential equation

$$y'' - xy' - x^2y = 0.$$