(10%) 1. Let $f:(0,+\infty)\to R$ be differentiable and $\lim_{x\to+\infty} f'(x)=5$.

Find $\lim_{x\to+\infty}(f(x+5)-f(x))$.

(20%) 2. Evaluate the following limit:

(5%) (a) $\lim_{x\to 0}\dfrac{e^x-1-x-\dfrac{x^2}{2}}{x^3}$.

(5%) (b) $\lim_{x\to 1^-}\dfrac{\dfrac{\pi}{2}-\sin^{-1}(x)}{x-1}$.

(5%) (c) $\lim_{x\to 0^+} x^{\frac{1}{2+\ln(x)}}$.

(5%) (d) $\lim_{x\to-\infty} x^2(1-x\sin(\frac{1}{x}))$.

(20%) 3. Evaluate the following integral:

(5%) (a) $\int_2^4 \frac{1+x}{1-x}dx$.

(5%) (b) $\int_{\pi/4}^{\pi/2}\frac{\cos^3(x)}{\sqrt{\sin(x)}}dx$.

(5%) (c) $\int_1^e \ln^2(x)dx$.

(5%) (d) $\int_0^2 \frac{1}{1+e^x}dx$.

(10%) 4. Let $\Gamma(\alpha)=\int_0^{+\infty} e^{-x}x^{\alpha-1}dx$, $\alpha\in(0,+\infty)$. Prove

(5%) (a) $\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$, $\forall\alpha>0$.

(5%) (b) $\Gamma(n)=(n-1)!$, $\forall n\in N$.

(20%) 5. Let $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$.

(5%) (a) Find A^{-1}.

(15%) (b) Find the eigenvalues of A and a non-singular matrix, P, such that $P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$.

(10%) 6. Show that if $A^2 = A$, and if λ is an eigenvalue of A, then either $\lambda = 1$ or $\lambda = 0$.

(10%) 7. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$. Find a symmetric matrix, B, such that $x^T A x = x^T B x$, $\forall x \in R^2$.