



- (1) Consider the 1st order O.D.E.

$$y + 4xy' = 0$$

Please find

- (a) the general solution by using separable method (10%);
(b) the integrating factor of the O.D.E. (10%);
(c) the unique solution that satisfies the initial condition $y(1) = 2$ (5%)
- (2) If $f(t)$ is a function defined for all $t \geq 0$, the Laplace transform of f is defined by

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

For all s such that this integral converges. Please find the Laplace transform of the following functions using the definition above.

- (a) $f(t) = \sin(2t)$ (10%)
(b) $f(t) = 3e^{-t}$ (15%)



(3) (25%)

Please find the tangent plane and normal line to the surface $z = x^2 + y^2$ at the point $(2, -2, 8)$.

(4) (25%)

Given $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, Please find

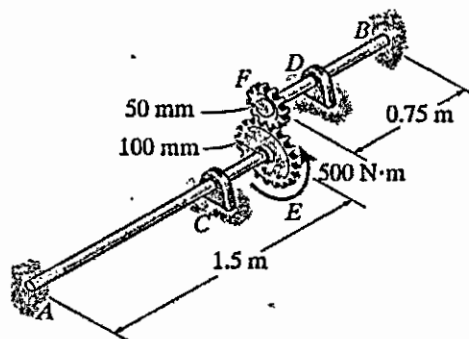
- (1) The eigenvalues of A .
- (2) A^N for a given positive integral N .



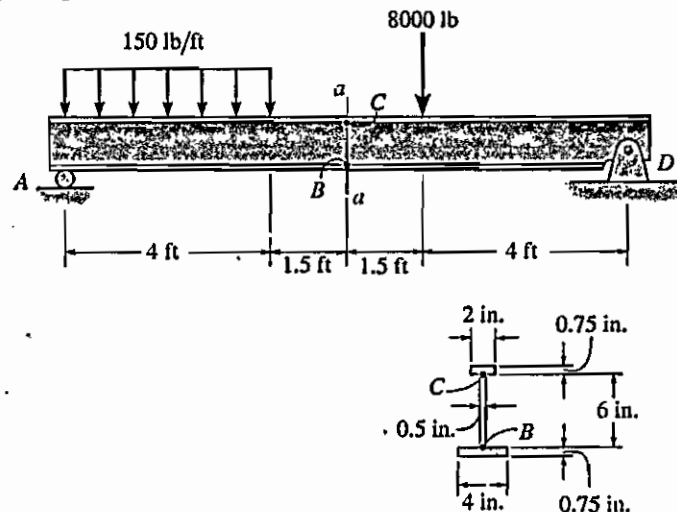
國立雲林科技大學 104 學年度
碩士班招生考試試題

系所：機械系
科目：材料力學

1. The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B . They are also supported by journal bearings at C and D , which allow free rotation of the shafts along their axes. If a torque of $500 \text{ N}\cdot\text{m}$ is applied to the gear at E as shown, determine the reactions at A and B . Determine the rotation of the gear at E . The shear modulus of A-36 steel is $G=75 \text{ GPa}$. [25%]



2. Determine the shear stress at points B and C on the web of the beam located at section $a-a$. [25%]





3. The member shown in Fig. 3 has a rectangular cross section.

- (a) Determine the reaction at point A. (5%)
 (b) Determine the reaction at point B. (5%)
 (c) Determine the state of stress that the loading produces at point C. (15%)

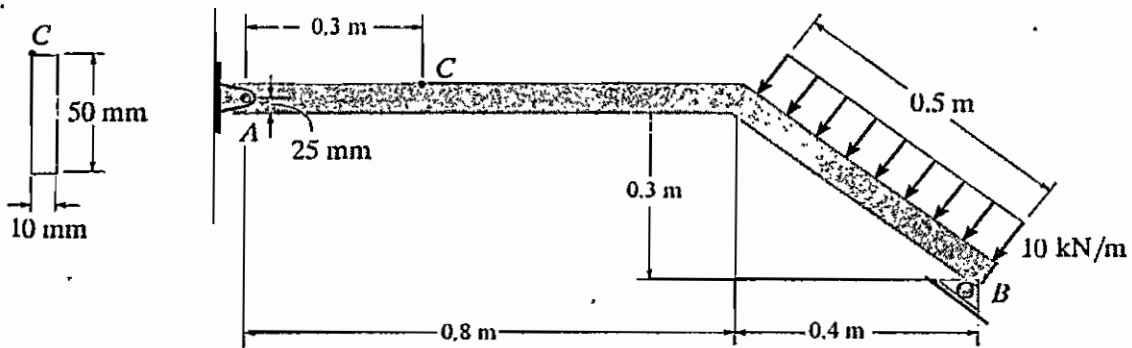


Fig. 3

4. The simply supported box beam shown in Fig. 4 supports the triangular distributed loading. If the material of the beam is linearly elastic and follows Hooke's law. The material's modulus of elasticity is 200 GPa.

- (a) Determine the moment in the beam as a function of x for $0 < x \leq 5$. (10%)
 (b) Determine its maximum deflection. (15%)

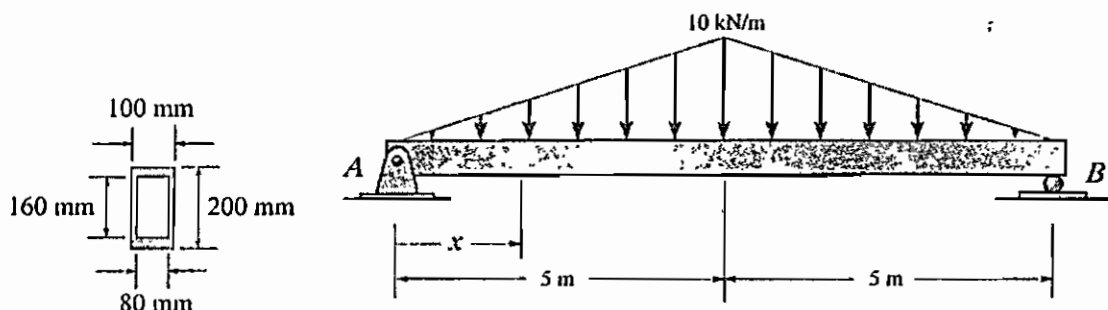


Fig. 4



1. (20%) For the system shown in Fig. 1a, the unit step response is plotted in Fig. 1b. Which of the following combinations is correct? Please **explain** your choice.

(A)

$$G_1(s) = \frac{4}{s^2 + 4s + 4} \quad G_2(s) = \frac{25}{s^2 + s + 25} \quad G_3(s) = \frac{100}{s^2 + 2s + 100}$$

(B)

$$G_1(s) = \frac{100}{s^2 + 2s + 100} \quad G_2(s) = \frac{25}{s^2 + s + 25} \quad G_3(s) = \frac{4}{s^2 + 4s + 4}$$

(C)

$$G_1(s) = \frac{25}{s^2 + s + 25} \quad G_2(s) = \frac{4}{s^2 + 4s + 4} \quad G_3(s) = \frac{100}{s^2 + 2s + 100}$$

(D)

$$G_1(s) = \frac{25}{s^2 + s + 25} \quad G_2(s) = \frac{100}{s^2 + 2s + 100} \quad G_3(s) = \frac{4}{s^2 + 4s + 4}$$

Fig. 1a

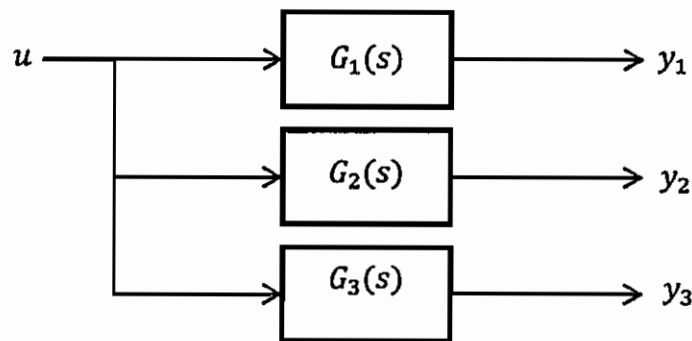
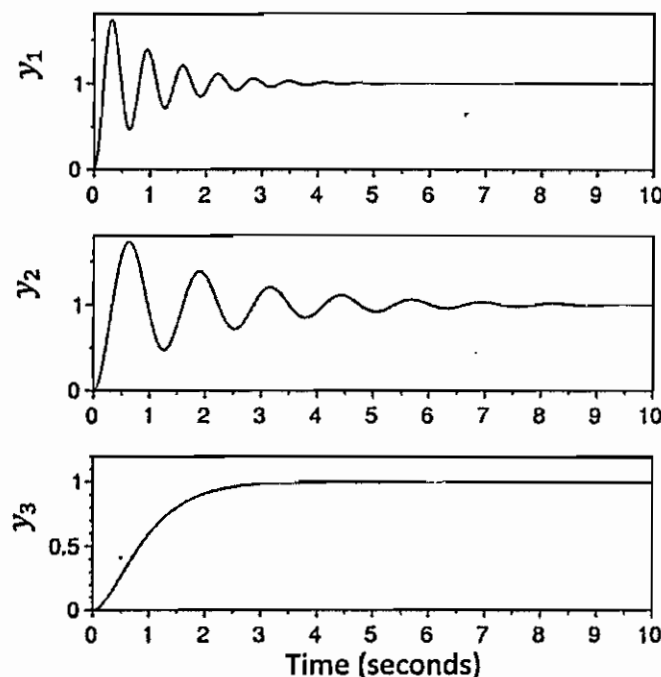


Fig. 1b

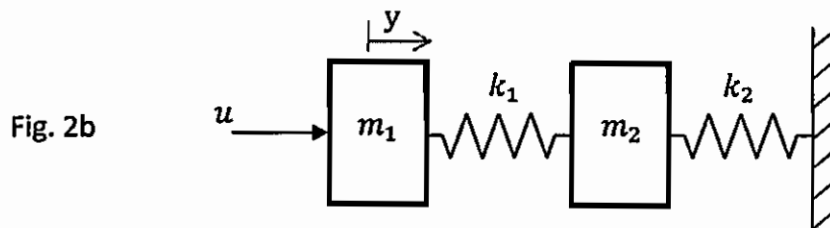
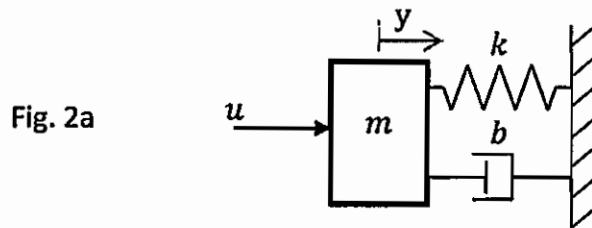




2. Determine the transfer function between y and u for the following systems.

(A) Find $\frac{Y(s)}{U(s)}$ for the system shown in Fig. 2a, where u is the force applied to the mass and y is the displacement of the mass. (8%)

(B) Find $\frac{Y(s)}{U(s)}$ for the system shown in Fig. 2b. (8%)



(C) Find $\frac{Y(s)}{U(s)}$ for the following system: (7%)

$$\ddot{x}_1 + 2\dot{x}_1 = x_2$$

$$\dot{x}_2 + x_2 = u$$

$$y = x_1$$

(D) Find $\frac{Y(s)}{U(s)}$ for the following system: (7%)

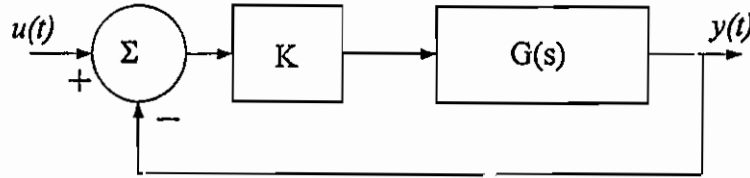
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



3. (total 50%)

A feedback control system is given as below:



The open loop transfer function $G(s)$ is a rational function and $G(s) = N_o(s)/D_o(s)$, where $N_o(s)$ and $D_o(s)$ are two polynomials. The root-locus plot is shown in figure 3a

- (1) If the control gain K changes from zero to infinite, what is the direction of each locus segment in which the closed loop roots move along its root locus? (6%) and please show why they should be? (8%)

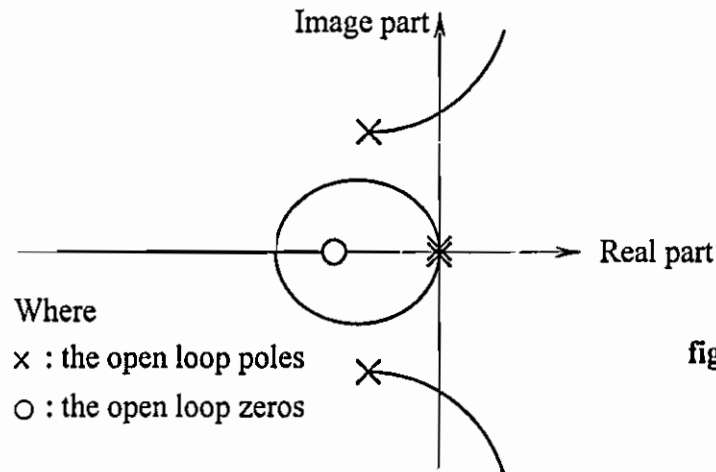


figure 3a

- (2) According to the information given in figure 3b, what is the stable range of control gain K if you apply the Routh's table and stability criterion to the given control system? (5%)

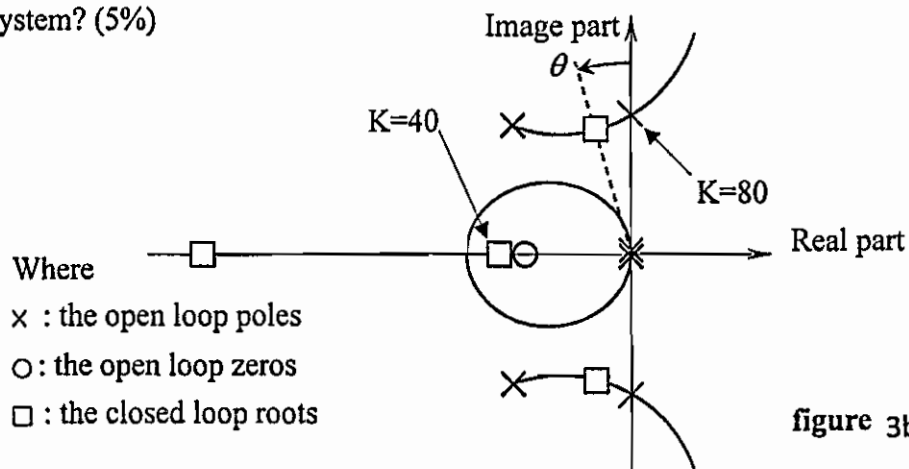


figure 3b



國立雲林科技大學 104 學年度
碩士班招生考試試題

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科目：自動控制

- (3) If the control gain $K=40$ is chosen as shown in figure 3b, is this control system stable? (3%) and please give the reason why it should be. (3%)
- (4) What is the damping ratio of this control system with the control gain $K=40$ as shown in figure 3b, which may be estimated from the dominant poles, where $\theta = 15^\circ$? (10%)
- (5) What is the gain margin (GM) in this control system if the control gain $K=40$ is chosen? (5%) Note that the unit of dB must be used in your answer.
- (6) How do you improve the performance of this control system to reduce the percentage of over shoot in its unit step response? (5%)
- (7) What is the steady state error for the input $u(t) = 10t$? (5%)



1. In addition to the customary horizontal velocity components of the air in the atmosphere, there often are vertical air currents shown in Figure 1. Assume that the velocity in a certain region is approximated by $u = u_0$, $v = v_0(1 - y/h)$ for $0 < y < h$, and $u = u_0$, $v = 0$ for $y > h$. Determine the equation for the streamlines and plot the streamline pass through the origin for values $u_0/v_0 = 0.5, 1.0, 2.0$. 25%

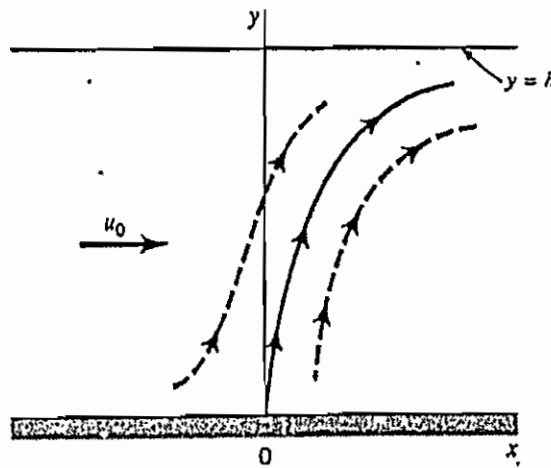


Figure 1.

2. Air flows steadily between two cross sections in a long, straight section of 0.25-m inside diameter pipe. The static temperature and pressure at each section are indicated in Figure 2. If the average air velocity at section (2) is 320 m/sec, determine the average velocity at section (1). 25%

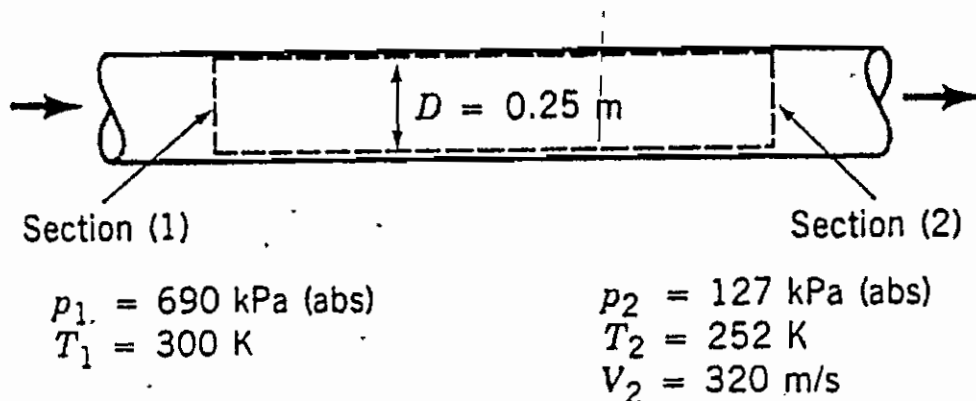
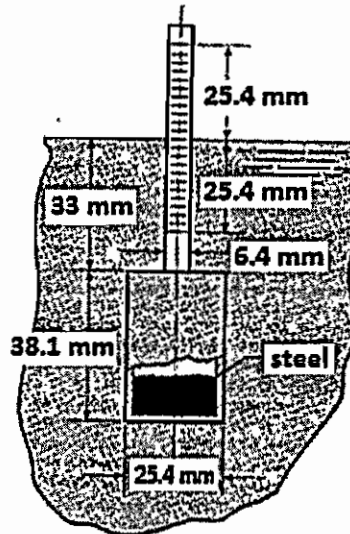


Figure 2.



3. (25%) A hydrometer is a device for indicating the specific gravity of liquids. The figure below shows the design for a hydrometer in which the bottom part is a hollow cylinder with a 25.4 mm diameter, and the top is a tube with a 6.4 mm diameter. The empty hydrometer weighs 0.088 N. a) What weight of steel should be added to make the hydrometer float in the position shown in fresh water? b) What will be the specific gravity of the fluid in which the hydrometer would float at the top mark?



4. (25%) For the system shown, kerosene ($sg=0.82$) is to be forced from tank A to reservoir B by increasing the pressure in the sealed tank A above the kerosene. The total length of 2-in Schedule 40 steel pipe is 38 m. Calculate the required pressure in tank A to cause a flow rate of 435 L/min. (the inner diameter of the 2-in steel pipe is 0.0525 m, friction factor f for the pipe is 0.0222, loss coefficients for entrance, check valve, angle valve, elbow and exit are 1, 1.9, 2.85, 0.57 and 1, respectively. Density of water is 1000 kg/m^3 and $g = 9.81 \text{ m/s}^2$)

