

考試科目	統計學 41823	所別	國際管理學院 學系管理組	考試時間	2月27日(六)第三節
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1. (20%) Let r_i , for $i = 1, \dots, n$, be independent samples of a random variable R of mean μ and variance σ^2 . Define the estimate

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n r_i$$

$$s^2 = \frac{1}{n} \sum_{i=1}^n (r_i - \hat{\mu})^2$$

Show that $E[s^2] = \sigma^2$.

2. (20%) Suppose a stock's rate of return has annual mean and variance of μ and σ^2 . To estimate these quantities, we divide 1 year into n equal periods and record the return for each period. Let μ_n and σ_n^2 be the mean and variance for the rate of return for each period. In particular, assume that

$$\mu_n = \frac{\mu}{n} \text{ and } \sigma_n^2 = \frac{\sigma^2}{n}$$

If $\hat{\mu}_n$ and $\hat{\sigma}_n^2$ are estimates of these, then $\hat{\mu} = n\hat{\mu}_n$ and $\hat{\sigma}^2 = n\hat{\sigma}_n^2$.

- (A) Show that the standard deviation of $\hat{\mu}$ is independent of n
 (B) Show how the standard deviation of $\hat{\sigma}^2$ depends on n
3. (20%) A coin having probability p of coming up heads is successively flipped until two of the most recent three flips are heads. Let N denote the number of flips. Find $E[N]$. Note that if the first two flips are heads, then $N = 2$.
4. (20%) Suppose that we continually roll a die until the sum of all throws exceeds 100. What is the most likely value of this total when you stop?
5. (20%) Suppose X is a Poisson random variable with mean λ . The parameter λ is itself a random variable whose distribution is exponential with mean 1. Compute the probability mass function $P(X = n)$

備

註

- 一、作答於試題上者，不予計分。
 二、試題請隨卷繳交。