

考試科目

統計學  
41833

所別

社會科學院

考試時間

2月27日(下) 第三節

## 2016 Mathematical Statistics Examination

1. (40%) Please explain the following items.

- (a) Sufficiency, Completeness and Stochastic Independence (8%)
- (b) Limiting distribution and Limiting Moment-generating function (8%)
- (c) Bayesian Estimates and Minimax Principle (8%)
- (d) Chebyshev's Inequality and the Rao-Cramer Inequality (8%)
- (e) The Uniformly Most Powerful Tests and Likelihood Ratio Tests (8%)

2. (15%) Given  $f(x; \theta) = \frac{1}{\theta}, 0 < x < \theta$ , zero elsewhere, with  $\theta > 0$ , formally compute the reciprocal of

$$100E\left\{\left[\frac{\partial \ln f(x; \theta)}{\partial \theta}\right]^2\right\}.$$

Compare this with the variance of  $\frac{101}{100}Y_{100}$  where  $Y_{100}$  is the largest item of a random sample of size  $n = 100$  from this distribution.

3. (10%) Let the joint p.d.f. of  $X$  and  $Y$  be  $f(x, y) = \frac{13}{7}x(x+y), 0 < x < 1, 0 < y < 1$ , zero elsewhere. Let  $U = \min(X, Y)$  and  $V = \max(X, Y)$ . Find the joint p.d.f. of  $U$  and  $V$ .
4. (10%) Let  $X_1, X_2, \dots, X_n$  be a random sample from the normal distribution  $N(\theta, 100)$ . Show the likelihood ratio principle for testing  $H_0 : \theta = 10$  against  $H_0 : \theta \neq 10$ .
5. (10%) The Pareto distribution is used as a model in study of claim losses and has the distribution function

$$F(x; \theta_1, \theta_2) = 1 - \left(\frac{\theta_1}{x}\right)^{\theta_2}, \theta_1 \leq x, \text{ zero elsewhere, where } \theta_1 > 0 \text{ and } \theta_2 > 0.$$

If  $X_1, X_2, \dots, X_{100}$  is a random sample from this distribution, find the maximum likelihood estimators of  $\theta_1$  and  $\theta_2$ .

6. (15%) Let  $X \sim b(n, p)$  and  $L(p, d(x)) = 2[p - d(x)]^2$ . Let  $\pi(p) = 1$  for  $0 < p < 1$  be the prior pdf of  $p$ . Then find (a) the posterior pdf of  $p$ , (b) the Bayes estimate of  $p$  and (c) the Bayes risk.

備

註

- 一、作答於試題上者，不予計分
- 二、試題請隨卷繳交。