考 試 科 目	數理統計學 'H4I2	所　別	統計學系	考 試 時 間	2 月 27 日（六）第 三 節

1. A recent study reports that the average time spent online is 7 hours per day.

(a) What is the probability that time spent online tomorrow will exceed 12 hours? (4%)

(b) If the variance of time spent on line per day is 16, what is the probability that time spent online tomorrow will be between 2 to 12 hours? (6%)

2. The joint distribution of pollutant standards index (PSI) collected under without using an air filter (X_1) and with using air filter (X_2) is

$$f(x_1, x_2) = \begin{cases} 1/4, & 0 \le x_1 \le 4, 0 \le x_2 \le 2, 2x_2 \le x_1 \\ 0, & \text{elsewhere} \end{cases}.$$

(a) Find $E(X_2 \mid x_1)$ and $V(X_2 \mid x_1)$. (10%)

(b) Let $W = X_1 - X_2$ be the reduction in PSI due to using an air filter. Find the probability density function of W, and E(W). (10%)

3. Let $X_1, \cdots, X_n$ be a. random sample from $f(x) = \dfrac{(\ln \beta)^x}{x! \beta}$, $x = 0, 1, \ldots$, $\beta > 1$.

(a) Find the complete sufficient statistic for β, and the MLE of β. (10%)

(b) Find the UMVUEs of $\ln \beta$ and $(\ln \beta)^2$, respectively. (6%,8%)

4. Let $X_1, \cdots, X_n$ be a random sample from a continuous distribution function $F(x)$, and $F'(x) = f(x)$. Let $X_{(k)}$ denote the kth order statistic.

(a) For some α, where $0 < \alpha < 1$, let x_α satisfy $F(x_\alpha) = \alpha$ and $f(x_\alpha) > 0$. Moreover, $m/n \to \alpha$.

(a1) Show that $F(X_{(m)})$ converges in probability to a constant and find that constant. (6%)

(a2) Find the limiting distribution of $\sqrt{n}(F(X_{(m)}) - \alpha)$. (6%)

(Hint: The limiting distribution of $\sqrt{n}(X_{(m)} - x_\alpha)$ is $N(0, \alpha(1-\alpha)/[f(x_\alpha)]^2)$.)

(b) Find the limiting distribution of $Y_n = n[1 - F(X_{(n)})]$. (10%)

5. Let $X_1, \cdots, X_n$ be a. random sample from $f(x) = \dfrac{\beta x^{\beta-1}}{\lambda} \exp(-\dfrac{x^\beta}{\lambda})$, $x > 0$, $\lambda > 0$, and β is known.

(a) Find the likelihood ratio size α test of $H_0 : \lambda = \lambda_0$ vs. $H_1 : \lambda \ne \lambda_0$. (10%)

(b) Find an equal tailed $100(1-\alpha)\%$ confidence interval for λ. (6%)

(c) Let the prior of λ be $\pi(\lambda) = \dfrac{b^a}{\Gamma(a)\lambda^{a+1}} \exp(-\dfrac{b}{\lambda})$, $\lambda > 0$, $a, b > 0$.

　　Find the posterior distribution and the posterior mean of λ. (8%)

備　　　註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。