

考試科目	基礎數學 41412	所別	統計學系	考試時間	之月27日(六)第一節
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- (10%) Find the rank of $A = \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$, where a, b , and c are real numbers.
- (10%) Let A, B , and C are matrices and \tilde{x}, \tilde{y} , and \tilde{z} are vectors. If $C^{-1} = A^{-1} + B^{-1}$ and $C^{-1}\tilde{z} = A^{-1}\tilde{x} + B^{-1}\tilde{y}$, then show that $\tilde{z} = \tilde{x} + (I + AB^{-1})^{-1}(\tilde{y} - \tilde{x})$.
- (10%) Find the inverse of the following matrix $\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$.
- (10%) Find all 2×2 matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $AB = BA$, where $B = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.
- (10%) Find the eigenvalues of matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$.
- (10%) Discuss $\lim_{x \rightarrow 0^+} \frac{\sin(x)}{|x|}$, $\lim_{x \rightarrow 0^-} \frac{\sin(x)}{|x|}$, and $\lim_{x \rightarrow 0} \frac{\sin(x)}{|x|}$.
- (7%) If $s_n = \frac{1}{2 \times 1} + \frac{1}{3 \times 2} + \dots + \frac{1}{(n+1) \times n}$, for $n = 1, 2, 3, \dots$, then $\lim_{n \rightarrow \infty} s_n = ?$
(7%) Prove that $\lim_{n \rightarrow \infty} n^{1/n} = 1$. (Hint: Binomial expansion on $(1 + a_n)^n$, where $a_n = n^{1/n} - 1$.)
- Compute the following integrals:
(5%) (a) $\int \frac{x^2}{\sqrt{1-x^3}} dx$
(5%) (b) $\int \cos(\pi t) \cos(\sin(\pi t)) dt$
- Compute the following derivatives:
(5%) (a) $\frac{d}{dx} \left(\int_0^x x^2 t^2 dt \right)$
(5%) (b) $\frac{d}{dx} \left(\int_1^x \cos(x+t) dt \right)$
- (6%) Find the Taylor (or polynomial) expansion for $x \cos(x)$.

備

註

- 作答於試題上者，不予計分。
- 試題請隨卷繳交。