東海大學105學年度碩士班考試入學試題

考試科目： 統計學C　　　　　　　　　　　　　科目代碼： 47112
應考系組： 統計系甲組
考試日期：105年03月06日第4節　　　使用計算機：可　　　　　共 2 頁(第 1 頁)

請於答案卷作答，違者不予計分

1. Two digits are chosen at random without replacement from the set of integers $\{1,2,3,4,5,6,7,8\}$.
(7%) (a) Find the probability that both digits are greater than 5.
(8%) (b) Show that the probability that the sum of the digits will be equal to 5 is the same as the probability that their sum will exceed 13.

2. Let X be a random variable with probability density function $f(x) = \frac{1}{2}e^{-|x|}$, $x \in (-\infty, \infty)$.
(7%) (a) Compute $P(-1 < X < 2)$.
(8%) (b) Find the probability density function of $Y = |X|$.

3. Let (X_1, X_2) be a random vector with joint probability mass function $f(x_1, x_2) = \frac{1}{4}$, $(x_1, x_2) = (0,0), (1,0), (0,1), (-1,-1)$.
(8%) (a) Compute the conditional probability mass function of X_2 given $X_1 = 0$.
(7%) (b) Find the probability mass function of $Y = X_1 + X_2$.

4. Let $X_1, \ldots, X_n$ be a random sample from uniform distribution over $[\theta-1, \theta+1]$, $\theta \in (-\infty, \infty)$, and $X_{(1)} = \min\{X_1, \ldots, X_n\}$ and $X_{(n)} = \max\{X_1, \ldots, X_n\}$.
(7%) (a) Is $(X_{(1)}, X_{(n)})$ a sufficient statistic for θ?
(8%) (b) Find the maximum likelihood estimator for θ.

5. Let $X_1, \ldots, X_n$ be a random sample from $Bernoulli(p)$, $p \in (0,1)$.
(7%) (a) Give an unbiased estimator for p^2.
(8%) (b) Find the uniformly minimum variance unbiased estimator (UMVUE) for p^2.

6. A sample of size 1 is taken from a population with probability density function $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, where $\theta > 0$.
(7%) (a) Find a most powerful (MP) size α of test $H_0 : \theta = 1$ against $H_1 : \theta = 2$.
(8%) (b) Find a uniformly most powerful (UMP) size α of test $H_0 : \theta \leq 1$ against $H_1 : \theta > 1$.

7. (10%) Show that $\binom{n}{k} p^k (1-p)^{n-k} \to e^{-\lambda} \frac{\lambda^k}{k!}$ as $n \to \infty$ and $p \to 0$ in such a way that $np = \lambda$ remains fixed.

作答前請先檢查試題頁數、頁次及考試科目是否正確，如有缺損或印刷不清等，應即舉手請監試人員處理，繳卷時試題須繳回。