| 考試科目 | 計算機數學 | 所別 | 資訊科學系碩士班 | 考試時間 | 2 月 26 日(六) 第 3 節 |
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# I 離散數學部分(60%)

(1). 選擇題 (20%)(單選不倒扣)：

1.1. Which of the following properties need not hold for a relation to be a partial order?

(a) transitivity (b) reflexivity (c) anti-symmetry (d) symmetry

1.2. How many equivalence relations are there on the set {1,2,3} ?

(a) 5 (b) 6 (c) 8 (d) 9

1.3. A simple graph is said to be regular if all of vertices have the same degree. Then which of the following graphs is not regular? Suppose n is an integer > 1.

(a) $C_n$ (a cycle graph with n vertices) (b) $K_n$ ( a complete graph with n vertices )

(c) $Q_n$ (A n-dimensional hypercube) (d) $K_{n,n}$ (a complete bipartite graph with n vertices in both partitions)

1.4. Which of the following relations is well-founded?

(a) $(Z, \le)$ (b) $(Q, \le)$ (c) $(R, \le)$ (d) $(N, \le)$

1.5. How many total orders are there on the set S = {1,2,4,8,3,9,72} which are compatible with the divisibility relation on S?

(a) 6 (b) 8 (c) 10 (d) 12

1.6. Let G =(V, T, S,P) be a context free grammar where N = {S} is the set of non-terminal symbols, T={0,1} is the set of terminal symbols, S is the start symbol and the set of productions P = {S → 1, S →0SS, S→S0S, S→SS0}. Then which of the following strings is derivable from the grammar?

(a) 01010 (b) 10010 (c) 11100 (d) 00011

1.7. Which of the following notations best describes the order of the function $(n^5+3n^3+2) / (7n^2 + 2n+100)$ ?

(a) $O(n)$ (b) $\Theta(n^2)$ (c) $\Theta(n^4)$ (d) $O(n^5)$

1.8. Which of the following propositions is not a tautology?

(a) $p\to(q\to p)$ (b) $(p\vee q)\wedge(\sim p \vee r)\to(q \vee r)$ (c) $(\sim p\to\sim q)\to(q\to p)$ (d) $(p\to q)\to(\sim p\to\sim q)$

1.9. If a planar graph G has 12 vertices, each of degree 3. Let e and r be the number of edges and regions, respectively, of G. Then what is the value of e + r ?

(a) 24 (b) 26 (c) 30 (d) 36

1.10 Let m and n be respectively the maximum and minimum number of nodes of all fully binary trees of height 8. The what is the value of m + n ?

(a) 528 (b) 511 (c) 513 (d) 530

| 備註 | 試題隨卷繳交 |
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請注意：背面還有試題。

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I 離散數學部分(續前頁)

計算與證明: (40%)

**(2).** [15%] Consider the following linear non-homogeneous recurrence relation:

$a_n = 4\,a_{n-1} - 4\,a_{n-2} + 3\text{x}2^n$

(a) What is the general solution of its associated homogeneous recurrence relation $a_n = 4\,a_{n-1} - 4\,a_{n-2}$?

(b) Find a particular solution for the above non-homogeneous recurrence relation.

(c) Combine the above results to find a solution of the non-homogeneous relation satisfying the initial condition: a0=1 and a1= 7.

**(3).** [15%] Consider the following recursive procedure for computing the Ackermann's function A(m,n):

```
A(m,n)   =   if (m=0) then return 2n
               else if(n = 0) then return 0
               else if( n=1) then return 2
               else return A(m-1, A(m, n-1)).
```

(a) Prove by induction that A(m,2) returns 4 if m is an integer $\geq 1$.

(b) How many times will A(-,-) be called if we invoke A(1,n) with an integer $n \geq 1$? Just give your answer and do not count the initial A(1,n) invocation.

(c) Explain briefly why A(m,n) will always terminate(run to end) if we call it by passing two non-negative integers.

**(4).** [10%] A set S is said to be *infinite* if there exists a sequence $x_0, x_1, x_2, \ldots$ of elements of S such that $x_i \neq x_j$ if $i \neq j$, or more formally if there exists a 1-1 mapping f:N→S from the set of non-negative integers N to S. Show that a given set S is infinite if and only if there is a 1-1 mapping g : S→S such that g(S) (i.e., the range of *g*) is a proper subset of S.

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II. 線性代數部分(40%)

(5). [15%]

Determine whether or not each of the following matrices is *symmetric*, that is, $A^T = A$, or *skew-symmetric*, i.e., $A^T = -A$:

$$(a)\quad A = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}, \qquad (b)\quad B = \begin{bmatrix} 0 & 4 & -3 \\ -4 & 0 & 5 \\ 3 & -5 & 0 \end{bmatrix}, \qquad (c)\quad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(6). [10%]

Let $u_1 = (1, 2, 4)$, $u_2 = (2, -3, 1)$, $u_3 = (2, 1, -1)$ in $\mathbf{R}^3$. Show that $u_1, u_2, u_3$ are orthogonal, and write $v$ as a linear combination of $u_1, u_2, u_3$, where: $(a)\quad v = (7, 16, 6)$, $(b)\quad v = (3, 5, 2)$.

(7). [15%]

Find the dimension and a basis of the solution space $W$ of each homogeneous system:

$$\begin{aligned} x + 2y + 2z - s + 3t &= 0 \\ x + 2y + 3z + s + t &= 0, \\ 3x + 6y + 8z + s + 5t &= 0 \end{aligned} \qquad \begin{aligned} x + 2y + z - 2t &= 0 \\ 2x + 4y + 4z - 3t &= 0 \\ 3x + 6y + 7z - 4t &= 0 \end{aligned} \qquad \begin{aligned} x + y + 2z &= 0 \\ 2x + 3y + 3z &= 0 \\ x + 3y + 5z &= 0 \end{aligned}$$

$$(a) \qquad\qquad\qquad (b) \qquad\qquad\qquad (c)$$

| 備註 | 試題隨卷繳交 |
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