

# 逢甲大學104學年度碩士班考試入學試題

編號：027 科目代碼：214

科目	微積分	適用系所	應用數學系	時間	100 分鐘
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※請務必在答案卷作答區內作答。

共 1 頁第 頁

## 一、是非題(40%，每格4分)：

1. If  $f(x) > 1$  for all  $x$  and  $\lim_{x \rightarrow 0} f(x)$  exists, then  $\lim_{x \rightarrow 0} f(x) > 1$ .
2. If  $f$  is continuous on  $[-1, 1]$  and  $f(-1) = 4$  and  $f(1) = 3$ , then there exists a number  $c \in (-1, 1)$  such that  $f(c) = \pi$ .
3. If  $f$  is continuous at  $a$ , then  $f$  is differentiable at  $a$ .
4. There exists a function  $f$  such that  $f(x) > 0$ ,  $f'(x) < 0$  and  $f''(x) > 0$  for all  $x$ .
5.  $\int_{-1}^1 \left( x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2} \right) dx = 0$ .
6. If  $\sum_{n=1}^{\infty} c_n 6^n$  is convergent, then  $\sum_{n=1}^{\infty} c_n (-2)^n$  is convergent.
7. If  $\{a_n\}$  is decreasing and  $a_n > 0$  for all  $n$ , then  $\{a_n\}$  is convergent.
8. If  $\sum_{n=1}^{\infty} a_n$  is convergent, then  $\sum_{n=1}^{\infty} |a_n|$  is convergent.
9. If  $f$  is continuous, then  $\int_{-\infty}^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_{-t}^t f(x) dx$ .
10. If  $\int_0^1 f(x) dx = 0$ , then  $f(x) = 0$  for  $0 \leq x \leq 1$ .

## 二、計算題 (60%)

1. (40 %) Evaluate the following
  - (a)  $\lim_{x \rightarrow 3} \left( \frac{x}{x-3} \int_3^x \frac{\sin t}{t} dt \right)$
  - (b)  $\lim_{n \rightarrow \infty} \frac{1}{n} \left[ \left( \frac{1}{n} \right)^9 + \left( \frac{2}{n} \right)^9 + \cdots + \left( \frac{n}{n} \right)^9 \right]$
  - (c) If  $f(x) = \ln x + \tan^{-1} x$ , find  $(f^{-1})'(\pi/4)$ .
  - (d)  $\int x \sec x \tan x dx$
  - (e) Differentiate  $y = \frac{(x+1)^4 \sqrt{x^2+1}}{(3x+2)^5}$ .
2. (10 %) If  $f(x) = \sin(x^3)$ . Find  $f^{(15)}(0)$ .
3. (10 %) Find  $\int_0^1 \int_{\sin^{-1} y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x} dx dy$ .