國立中正大學 104 學年度碩士班招生考試試題

系所別：經濟學系國際經濟學-乙組　　科目：微積分

1. Determine whether the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ is convergent or divergent. **(8 pts.)**

2. Find $\lim_{x \to 0} \frac{x^2 \sin(\frac{1}{x})}{\tan x}$. **(8 pts.)**

3. Let $u_1 : \mathbb{R}_{++} \to \mathbb{R}$ with $u_1(W) = \frac{1-\gamma}{\gamma}\left(\frac{aW}{1-\gamma} + b\right)^{\gamma}$, where $a, b > 0$ and $\gamma \neq 1$, and let $u_2 : \mathbb{R}_{++} \to \mathbb{R}$ with $u_2(W) = \ln(W)$. Find $-\frac{u_i'(W)}{u_i''(W)}$ for each $i \in \{1, 2\}$. **(12 pts.)**

4. Let $g : \mathbb{R} \to \mathbb{R}$ with $g(x) = \int_{-x}^{ax+b} f(x, y)dy$, where $a, b > 0$ and f is a differentiable function on $\mathbb{R}^2$. Find $g'(x)$. **(8 pts.)**

5. Suppose that $\frac{x^3}{3} + \frac{y^2}{2} = xy$. Find $\frac{dy}{dx}$. **(8 pts.)**

6. Evaluate the following integrals:

   (1) $\int \sqrt{2 - e^x}dx$. **(8 pts.)**

   (2) $\int_{-\infty}^{0} (3 - x)e^{2x}dx$. **(8 pts.)**

   (3) $\int_0^2 \int_{y/3}^{1} \frac{y}{2} e^{x^3} dxdy$. **(8 pts.)**

7. Let $C : \mathbb{R}_{++}^2 \to \mathbb{R}$ with $C(S, K) = S \cdot N(f(S, K)) - e^{-rt}K \cdot N\left(f(S, K) - \sigma\sqrt{t}\right)$, where $r, t, \sigma > 0$, $N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$, and $f(S, K) = \frac{rt + \frac{\sigma^2 t}{2} - \ln\left(\frac{K}{S}\right)}{\sigma\sqrt{t}}$.

   (1) Find $N'(x)$. **(8 pts.)**

   (2) Find $\frac{\partial C(S,K)}{\partial S}$. **(8 pts.)**

   (3) Find $\frac{\partial C(S,K)}{\partial K}$. **(8 pts.)**

   (4) Show that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = 1$. **(8 pts.)**