國立中正大學104學年度碩士班招生考試試題

系所別：通訊工程學系-通訊丙組　　　　科目：機率

第1節　　　　

1. (10%)
Let X be a random variable with $\mathrm{E}\{X\}=1$ and $\mathrm{VAR}\{X\}=2$. Consider a new random variable Y defined as $Y=g(X)=aX+b$, where a and b are two constants. How can we choose the values for a and b so that $\mathrm{E}\{Y\}=2$ and $\mathrm{VAR}\{Y\}=1$?

2. (10%)
Earthquakes greater than Richter Magnitude 5.0 in Taiwan occur according to a Poisson Process of rate $\lambda=6$ times per year. Conditioned on the event that exactly 7 such earthquakes have occurred in 1999. What is the probability that exactly 2 such earthquakes occurred in the first half year?

3. (20%)
Let X be a random variable with the following probability density function (pdf):

$$f_X(x)=\begin{cases} 2xe^{-x^2} & , 0<x<\infty \\ 0 & , \text{otherwise.} \end{cases}$$

Let $Y=X^2+1$.

a) (10%) Please write down the pdf $f_Y(y)$ for the random variable Y.
b) (10%) Please use your result in part a) to show that $\int_{-\infty}^{\infty} f_Y(y)=1$.

4. (10%)
Consider a pair of random variables (X,Y) which is drawn uniformly within the rectangular region $\mathcal{R}=\{(x,y)|1\le x\le 2,\ 1\le y\le 3\}$. Let the new random variable Z be defined as $Z=\max\{X,Y\}$. Find the probability $P[Z\le 2.5]$.

5. (15%)

Independent Bernoulli trials, each of which is a success with probability p, are performed until the kth success.

a. (10%) What is the probability mass function (pmf) of necessary trials.
b. (5%) What is the mean number of necessary trials.

6. (15%)

Let X,Y have a joint pdf $f_{X,Y}(x,y)=6e^{-(2x+3y)}, x\ge 0, y\ge 0$

a. (5%) Are X and Y independent? Prove or disprove it.
b. (10%) Find the probability $P[X\geq 3Y]$.

7. (20%)

A die is tossed and the number of dots X is noted; and integer Y is then selected at random from $\{1,\ldots\ldots,X\}$

a. Find $P[Y=2]$.
b. Find $P[X=4|Y=2]$.