1. (15%) Let $\boldsymbol{R}^4$ have the Euclidean inner product. Find two unit vectors that are orthogonal to all three of the vectors $\mathbf{u} = (1,1,-4,0)$, $\mathbf{v} = (-1,1,2,-2)$, $\mathbf{w} = (3,-2,5,4)$.

2. (15%) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, show that A is diagonalizable if $(a-d)^2 + 4bc > 0$.

3. (5%) Let $\boldsymbol{B}\mathrm{x} = \mathrm{b}$ be a linear system whose augmented matrix $(\boldsymbol{B}|\mathrm{b})$ has reduced row echelon form

$$\left(\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Find all solutions to the system.

4. (15%) Find a matrix S such that $S^2 = A$, if

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{bmatrix}$$

5. $d_1 = (1, 1, 1, 1)$, $d_2 = (1, 2, 3, 4)$, $d_3 = (0, 1, 0, 1)$, $d_4 = (1, 0, 1, 0)$, $d_5 = (4, 3, 2, 1)$.
   a. (5%) Find the dimension of the space V spanned by the vectors d_1, d_2, d_3, d_4, and d_5.
   b. (5%) Find a subset of these five vectors that forms a basis for the space V.
   c. (5%) Express each vector d_i not in the basis (found in 5(b)) as a linear combination of the basis vectors.

6. Two matrices M_1 (3 by 3) and M_2 (4 by 4) have the same determinant value.

$$M_1 = \begin{bmatrix} 4 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 3 & 4 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & m & 0 & 0 \\ m & 0 & m & 0 \\ 0 & m & 0 & m \\ 0 & 0 & m & 0 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 0 & m & 0 & 0 \\ 2m & 0 & 2m & 0 \\ 0 & 3m & 0 & 3m \\ 0 & 0 & 4m & 0 \end{bmatrix}$$

a. (5%) Find the value of m.
b. (5%) Find the determinant of the matrix M_3 (4 by 4).
c. (5%) Find the dimension of column space determined by M_3.
d. (10%) Determine the inverse matrix of M_3.

7. (10%) Let L be the linear transformation defined by $L(a + bx) = a + (a + b)x + bx^2$. Find the matrix representing L with respect to the ordered bases $\{1, x\}$ and $\{x^2, x + x^2, 1 + x^2\}$.