

1. (10%)

An urn contains three black balls and four white balls. Three balls are selected at random from the urn **without replacement**. Find the probability of getting two black balls and one white ball.

2. (10%)

Consider a communications system where the transmitter sends a message to the receiver through a noisy channel. The message is encoded so that the receiver can detect when errors have been introduced into the message during transmission. If the receiver detects an error, it requests the transmitter to retransmit. If the probability of a message transmission error is ε , what is the probability that a message needs to be transmitted more than k times? You can assume $k \geq 1$ in this question.

3. (10%)

Consider a random variable X which has the following probability density function (pdf):

$$f_X(x) = e^{-\pi(x-1)^2}, \quad -\infty < x < \infty.$$

Please express the cumulative distribution function (CDF) of X using Q-function.

4. (10%)

The random variable X and Y have a joint pdf given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{x} e^{-2x} & , \quad 0 \leq y \leq x < \infty \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Find the expected value of X .

5. (10%)

Please prove the memoryless property

$$P[M \geq k + j | M > j] = P[M \geq k], \quad \text{for all } j, k > 1$$

of a geometric random variable.

6. (20%)

Let U and V be two random variables and $X = U + aV$, $Y = bU + V$, where a and b are constants.

- Find an expression for $E\{XY\}$ in terms of joint moments of U and V .
- if U and V are independent standard Gaussian random variables, find the joint and marginal probability density functions (pdf) of X and Y .

7. (30%)

The lifetime of a cheap light bulb, X , is an exponential random variable with mean 10 hours, i.e., with pdf $f(x) = 10e^{-10x}$, $x \geq 0$. Suppose that 5 bulbs are installed at the same time, i.e., time 0.

- Find the pdf of the duration that at least one bulb is working.
- Find the probability that exact one bulb is working at time 10.
- Assume that at time 5, all of the three bulbs are still working. Find the probability that exact one bulb is working at time 10.