國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【醫科所碩士班選考】　　題號：428001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. Let matrix

$$A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$$

(a) Find the eigenvalues and the corresponding eigenvectors of the matrix A. (10%)
(b) Find matrix P that makes A diagonal, that is, $D = P^{-1}AP$ where D is a diagonal matrix. (5%)
(c) Find the n-th power of matrix A, that is, A^n. It is not necessary to explicitly compute P^{-1}. (5%)

2. Solve $x^3y''' - 2xy' + 4y = \ln x + x^3$ (5%)

3.Find the area and volume if the vertices are (1,1,1), (2,1,1), (1,5,1), (1,1,3). (10%)

4. Please use Laplace transform to solve the following initial value problem. (5%)

$$y''(t) + 3y'(t) + 2y(t) = 0,\ y(0) = 1,\ y'(0) = 0$$

5. For a system of non-linear ordinary equations (with m>0) in a two dimension phase plan.

$$y_1' = y_2 - 2$$

$$y_2' = \frac{2m}{\pi} y_1 - \sin y_1$$

(a) For m = 1, please find all the critical points in the phase plane. (10%)
(b) Find the range for the value of m such that this system of ordinary differential equation has seven values. (10%)

6. Let

$$A = \begin{pmatrix} 1 & 1/3 & -2/3 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & -1/6 & 1/3 & 0 & 0 \\ -1/2 & -2/3 & 1/3 & 1/2 & 0 \\ 0 & -1/3 & 0 & 0 & 1/3 \end{pmatrix} \quad \text{and } b = \begin{pmatrix} 5 \\ 5 \\ 1 \\ 7 \\ 8 \end{pmatrix}$$

(a) Solve for x such that $Ax = b$. (6%)
(b) Find all eigenvalues of A, and list them in descending order. (6%)
(c) Find a 5×5 matrix X and X^{-1} such that $X^{-1}AX = D$, where D is the diagonal matrix whose diagonal elements are eigenvalues of A in descending order. (6%)
(d) Find the determinant of A and the inverse of A. (6%)
(e) Find e^A. (6%)

7. Find the Fourier transform of $f(t) = t^2 e^{-5|t|}$ (10%)