國立嘉義大學 104 學年度

電機工程學系碩士班招生考試試題

科目：工程數學

(注意事項：1.不可使用計算機。2.依次序作答。3.試題隨試卷繳回。)

1. Determine the values of λ for which the following system of equations has nontrivial solutions. (10%)

$$\begin{array}{rcrcrcl} (5-\lambda)x_1 & + & 4x_2 & + & 2x_3 & = & 0 \\ 4x_1 & + & (5-\lambda)x_2 & + & 2x_3 & = & 0 \\ 2x_1 & + & 2x_2 & + & (2-\lambda)x_3 & = & 0 \end{array}$$

2. The three vectors $(3, 2, 0)$, $(1, 5, -1)$, and $(5, -1, 2)$ form a basis for $\mathbf{R}^3$. Use these vectors in the Gram-Schmidt process to construct an orthonormal basis for $\mathbf{R}^3$. (15%)

3. Consider the bases $B = \{(1, 0), (0, 1)\}$ and $B' = \{(1, 2), (-1,-1)\}$ of $\mathbf{R}^2$. If $\mathbf{u}$ is a vector such that $\mathbf{u}_B = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$, find $\mathbf{u}_{B'}$. (10%)

4. Determine the inverse of the following 4×4 matrix, if it exists, using the method of **Gauss-Jordan elimination**. $\begin{bmatrix} -3 & -1 & 1 & -2 \\ -1 & 3 & 2 & 1 \\ 1 & 2 & 3 & -1 \\ -2 & 1 & -1 & -3 \end{bmatrix}$ (15%)

5. Evaluate the following problems

(a) Find the Laplace transform of $f(t) = t^2 u(t-3)$. (5%)

(b) Find the Laplace transform of $f(t) = t(\cos t + \sin t)$. (5%)

(c) Find the inverse Laplace transform of $F(s) = \dfrac{s}{(s-2)(s-3)(s-6)}$. (5%)

6. Solve $\left[\sin(xy) + xy\cos(xy) + ye^x \right]dx + \left[x^2\cos(xy) + e^x \right]dy = 0$. (10%)

7. Solve the differential equation $y'' = 3 - (y')^2$. (10%)

8. Solve for currents in the circuit of Fig. 1, assuming that the currents and charges are initially zero and that $E(t) = 2u(t-4) - u(t-5)$, where $u(t)$ is unit step function. (15%)
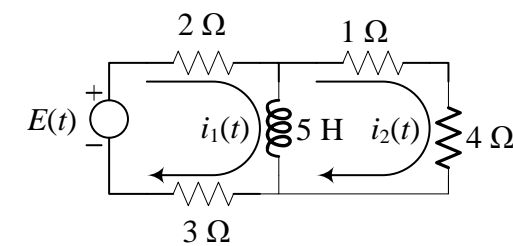

Fig. 1