國立臺灣海洋大學 101 學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目：計算機數學（含線性代數、離散數學）

系所名稱：資訊工程學系碩士班不分組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

Part I(50%)

1. (15%) Let Q_n be the n-cube in the space.

(a) How many vertices in Q_n?

(b) Derive how many edges in Q_n?

2. (20%) True or False, any n+1 integers from {1,2,……,2n} there must be an integer that divides one of the other？

3. (15%) A group of 10 students is assigned seats for each of 2 classes in the same classroom. How many ways can these seats be assigned if no student is assigned in the same seat for both classes?

Part II(50%)

1. (10%) Let the matrix A be

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

(a) Find the eigenvalues and corresponding eigenvectors of matrix A.
(b) Is A diagonalizable? If so, find a matrix P that diagonalizes A, and determine $P^{-1}AP$. (Hint: You do not need to find P^{-1}).

2. (10%) Let W be the subspace of $\mathbb{R}^6$ spanned by the vectors:

$$w_1 = (1, -2, 0, 4, 5, -3), \quad w_2 = (3, -7, 2, 0, 1, 4),$$
$$w_3 = (2, -5, 2, 4, 6, 1), \quad w_4 = (4, -9, 2, -4, -4, 7).$$

(a) Find a basis for the subspace W that is a subset of $\{w_1, w_2, w_3, w_4\}$.
(b) Find a basis for the orthogonal complement of W (i.e., $W^{\perp}$).

3. (15%) Suppose that a linear system $A\mathrm{x} = \mathrm{b}$ is expressed as

$$\begin{bmatrix} 3 & 10 & -10 \\ -2 & -4 & 11 \\ -1 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}$$

(a) Find the LU-decomposition (or factorization) of the coefficient matrix A, that is $A = LU$, where L is a lower triangular matrix and U is an upper triangular matrix with 1 in diagonal (7%).
(b) Using the result of (a) to solve the above system $A\mathrm{x} = \mathrm{b}$. (Hint: Let $U\mathrm{x} = \mathrm{y}$ and solve $L\mathrm{y} = \mathrm{b}$ first for y, and then solve x.)

4. (15%) Let $\mathbb{R}^3$ have the Euclidean inner product.
(a) verify that the vectors $\mathrm{u}_1 = (1, -1, 0)$, $\mathrm{u}_2 = (2, 0, -2)$, and $\mathrm{u}_3 = (3, -3, 3)$ form a basis for $\mathbb{R}^3$ (4%).
(b) Use the *Gram-Schmidt process* to transform the basis $B = \{\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3\}$ into an *orthonormal basis*, say $B' = \{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ (7%).
(c) Find the QR-decomposition of the matrix A, where A is the matrix with u_1 u_2, u_3 as column vectors (4%) (Hint: Using the results of (b)).