本試卷計 9 大題，合計 100 分

1. Evaluate $\oint_C (3y - e^{\sin x})dx + (7x + \sqrt{y^4 + 1})dy$, where C is the circle $x^2 + y^2 = 9$.

   (10%)

2. Find the general solution to $y' - 1 = e^{-y}(x-1)$. (10%)

3. Evaluate the integral $\int_{|z - 1/2| = 1} \frac{e^z}{z^3 - z}dz$. (10%)

4. Let a and k be positive numbers, and let (10%)

$$f(t) = \begin{cases} k & \text{for } -a \le t < a \\ 0 & \text{for } t < -a \text{ and for } t \ge a \end{cases}$$

   Find the Fourier transform of $f(t)$.

5. Find the general solution to $x^2 y'' + 2xy' - 12y = \sqrt{x}$, $x > 0$. (10%)

6. Find the Laplace-transforms of the following functions. (10%)

   (a) $y(t) = 5te^{-5t} + \frac{\sin(t)}{t}$

   (b) $f(t) = \frac{1}{2} + \frac{1}{2}e^{-2t} - e^{-t} - \frac{1}{2}e^{-t}t^2$

7. Find the inverse Laplace of the following functions. (20%)

   (a) $F(s) = \frac{1}{s^3 + 4s^2 + s - 6}$

   (b) $G(s) = \frac{10}{(s+1)^2(s+3)}$

8. Find (a) the characteristic equation　(b) the eigenvalues and corresponding eigenvectors of the following matrix. (10%)

$$A = \begin{bmatrix} 1 & 1 & -4 \\ 2 & 0 & -4 \\ -1 & 1 & -2 \end{bmatrix}$$

9. Find the rank and nullity of A. (10%)

$$A = \begin{bmatrix} 0 & 1 & 5 & 2 & -1 \\ 1 & 2 & 3 & 4 & -1 \\ 2 & 0 & 2 & 0 & 2 \end{bmatrix}$$