

國立臺灣海洋大學 101 學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目：微分方程與線性代數

系所名稱：電機工程學系碩士班控制組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. (20 %) Find the singular value decomposition of the matrix

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

2. (15%) Show that a complex hermitian matrix A is positive definite if and only if all its eigenvalues are real and positive.

3. (15%) Find the minimal polynomial of the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 4 & -2 \\ 0 & 3 & -1 \end{bmatrix}$$

4. (10%) Find the general solution for the differential equation

$$y'' - 6y' + 9y = e^{3x} + 1.$$

5. (10%) Find the general solution for the differential equation

$$xy' = \frac{x^2}{y} + y.$$

6. (15%) Solve the initial value problem

$$y'' - 4y' + 4y = f(t); y(0) = -2, y'(0) = 1, \text{ with } f(t) = \begin{cases} t, & \text{for } 0 \leq t < 3 \\ t+2, & \text{for } t \geq 3 \end{cases}$$

7. (15%) Let $A = \begin{bmatrix} -5 & 0 & 6 \\ 0 & 1 & 0 \\ -3 & 0 & 4 \end{bmatrix}$. (a) Find e^{At} . (b) Find the general solution for

$$x'(t) = Ax(t) + g(t), \text{ where } g(t) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$