招 生 學 年 度	104	招 生 類 別	碩士班
系 所 班 別	應用數學系 統計碩士班		
科 目 名 稱	基礎數學		
注 意 事 項	本考科禁止使用掌上型計算機；含微積分及線性代數		

You must show your work for each answer.

1. (10 points) Find the value of $\lim_{x \to 0^+} x^x$.

2. (10 points) Find the derivative of $f(x) = x^5 \ln(e^{5x} + 1)$.

3. (10 points) Find the area of the region enclosed by the graphs of $y = (1/4)x^2$ and $y = 8/(x^2 + 4)$.

4. Let $f(x) = \ln(x)$.

(a) (10 points) Find the fourth-degree Taylor polynomial $P_4(x)$ of $f(x)$ at $c = 1$.

(b) (10 points) Find the maximum error incurred if $f(1.1)$ is approximated by $P_4(1.1)$.

5. (10 points) Let $A = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$. Find the inverse of the matrix A.

6. Let W be the subspace of R^4 spanned by the three vectors: $\alpha_1 = (1, 2, 2, 1)$, $\alpha_2 = (0, 2, 0, 1)$, and $\alpha_3 = (-2, 0, -4, 3)$.

(a) (10 points) Prove that the three vectors α_1, α_2, and α_3 are independent.

(b) (10 points) Is the vector $\beta = (4, -5, 9, -7)$ in the subspace W?

7. Let $C = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$.

(a) (10 points) Find all eigenvalues of the matrix C.

(b) (10 points) Find one eigenvector for each eigenvalue of the matrix C.