1. (30%) A physical system is described by the following ordinary differential equation, where $y(t)$ is the output, $u(t)$ is the input and the coefficients $K_1, K_2 > 0$:

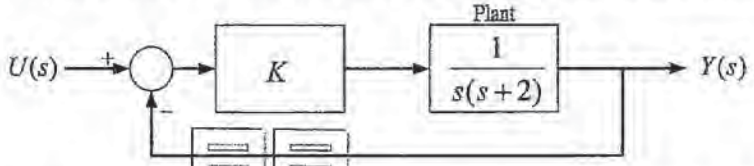
$$2y'''(t) + y''(t) + K_1 y'(t) + y(t) = K_2 u(t).$$

(a) Determine the input-output relationship of this system with both transfer function and state variable models. (10%)

(b) Describe the relationship of these two models and compare their difference. (10%)

(c) Determine the ranges of K_1 and K_2 for which this system is stable. (10%)

2. (20%) The 2nd-order control system with unit-step input is shown as follows:



(a) Determine the values of control gain K so that the system can be classified to be **(i)** underdamped, **(ii)** overdamped, or **(iii)** critically damped, if possible. (10%)

(b) Find the output response of this system if $K = 1$. (10%)

3. (30%) A system has the state variable model in $\dot{x} = \begin{bmatrix} 0 & 3 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ and $y(t) = x_1$.

(a) Is this system bounded-input-bounded-output stable? Explain your answer. (10%)

(b) Determine an equivalence transformation $\bar{x}(t) = Px(t)$ that transforms the given state equations into a diagonal canonical form, if possible. (10%)

(c) Find the zero-state response of this system to a unit step input. (10%)

4. (20%) Consider the following figure of root locus from $K = 0$ to $K = \infty$.

(a) Determine the value of gain K resulting in a marginally stable system. (10%)

(b) With addition of an extra pole at $s = -1$ in the following figure, determine the value of gain K for which the closed-loop transfer function will have a pole on the real axis at $s = -3$. (10%)