1. Solve the initial value problem **(16%)**

$$e^{x+y}y' = 3x \; ; \quad y(-1) = 1.$$

2. Find the general solution of the following equation **(16%)**

$$x^2y'' + 2xy' - 6y = x^2 - 2.$$

3. Use the <u>Laplace</u> transform to solve the initial value problem **(16%)**

$$y'' + 9y = t^2 \; ; \quad y(0) = y'(0) = 0.$$

4. In the following equation, (a) use the method of Frobenius to find the indicial equation and obtain r_1 and r_2; (b) determine the first solution $y_1(x)$. **(20%)**

$$4x^2y'' - 3xy' + 4y = 0.$$

　參考公式　Frobenius series: $y(x) = \sum_{n=0}^{\infty} C_n(x - x_o)^{n+r}$

5. Find the eigenvalues and the corresponding eigenfunctions of the following problem on an interval $[0,\pi]$. **(16%)**

$$y'' + \lambda y = 0 \; ; \quad y'(0) = y'(\pi) = 0.$$

6. Find the Fourier series of the function on the interval. **(16%)**

$$f(x) = \cosh(x), \quad \text{for} \quad -1 \leq x \leq 1.$$