1. Express data using the representation methods described below:
   a) Express the fractional number 29/7 in 8-bit fixed-point binary notation. The radix point is set at the middle of the 8-bit notion. (4%)
   b) Express the floating point number 1.63 in binary notation using 1-bit sign, 3-bit exponent in excess-4 notation (100 stands for 0), and 4-bit mantissa. (6%)

2. Design a combinational circuit that takes an unsigned 2-bit number $x = [x_1\ x_0]$ and computes the square of x, i.e., $y = x^2$. Here y is the output of the calculation.
   a) How many bits do you need for the output y? (1%)
   b) Show the truth table for this combinational circuit. Name the output $y = [y_n \ldots y_1\ y_0]$, where n is the number of bits determined in a). (5%)
   c) Determine the minimized output functions, i.e., the relationship between x_n and y_n. (4%)

3. Even parity refers to a parity checking approach in which a parity bit is set to one if there is an even number of one bits in an N-bit data item. If the number of one bits adds up to an odd number, the parity bit is set to zero. Answer the following questions based on this definition.
   a) Fill in the missing entries in this truth table for the 3-variable even parity function. (3%)

x	y	z	Even parity
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

   b) Write down the sum-of-products form of even parity for 3 variables. (3%)
   c) Can this truth table be simplified by using the Karnaugh map? Show your work. (4%)

4. Design a circuit that counts the number of 3 binary inputs A, B and C. The circuit output is a 2-bit number X_1 and X_0, counting the numbers of inputs that are logic HIGH.
   a) Find the minimized logic equations for outputs X_1 and X_0. Present X_0 using XOR operators. (6%)
   b) Draw the logic diagram using AND, OR, and XOR gates for this circuit. (4%)

5. Show that the following grammar is ambiguous: (10%)

$$S \rightarrow aSbS \mid bSaS \mid \lambda$$

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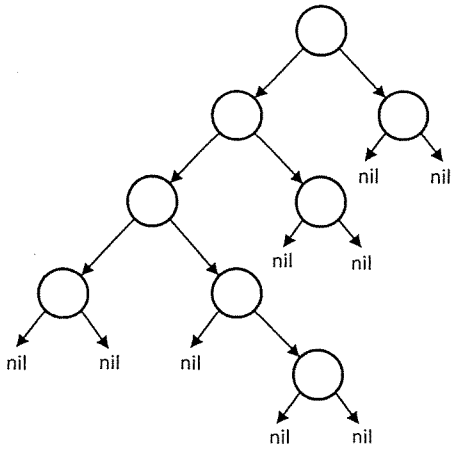
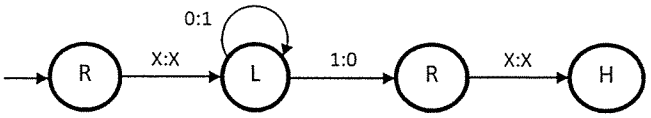
6. Binary-search-tree and red-black tree.
   a) Assign the keys 2, 3, 5, 7, 11, 13, 17, 19 to the nodes of the binary search tree below so that they satisfy the binary-search-tree property. (5%)

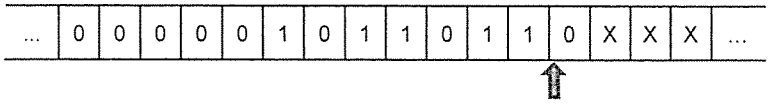


   b) The binary search tree can be transformed into a red-black tree by performing a single rotation. Draw the resulted red-black tree. Label each node with "red" or "black". Include the keys from part a). (5%)

7. The Turing machine below starts in the leftmost state.



   a) Suppose this machine is run on the tape below, with the tape head starting at the position marked by the arrow. What will be the contents of the tape when it halts? Where will the head be? (6%)



   b) What computation does this machine perform? (4%)

8. Suppose you run the following assembly program. Consider the operations and controls for the assembly.

```
10: 7201    R[2] <- 01
11: 7301    R[3] <- 01
12: 8115    R[1] <- Mem[15]
13: C117    if (R[1] == 0)  PC <- 17
14: 2112    R[1] <- R[1] - R[2]
15: 1832    R[3] <- R[3] + R[2]
16: C013    PC <- 13
17: 0000    Halt
```

```
OPERATIONS AND CONTROLS
1: add R[d] <- R[s] + R[t]
2: subtract R[d] <- R[s] - R[t]
3: and R[d] <- R[s] & R[t]
4: shift left R[d] <- R[s] << R[t]
5: shift right R[d] <- R[s] >> R[t]
6: halt halt
```

   Answer the questions below. Write your answers in the format of 4-digit hexadecimal numbers.
   a) What is the value of R[1] after the instruction at location 12 completes? (3%)
   b) What is the value of R[3] after the first time the instruction at location 13 completes? (2%)
   c) What is the value of R[1] when the program halts? (2%)
   d) What is the value of R[3] when the program halts? (3%)

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9. Consider the following program that is supposed to read an integer N and N strings from standard input, and print them to standard output in reverse order.

```
1  public class ReverseInputBuggy
2  {
3      public static void main(String[] args)
4      {
5          int N = StdIn.readInt();
6          String s;
7          for (int i = 1; i < N; i++)
8              s[i] = StdIn.readString();
9          for (int i = N; i >= 0; i--)
10             System.out.println(s[i]);
11     }
12 }
```

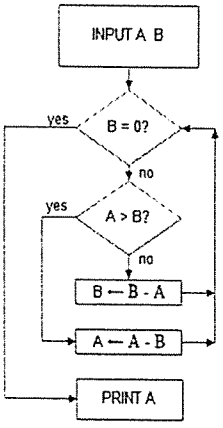
Answer the questions below.

a) Which bug prevents the program from compiling successfully? Identify the line number where the bug appears and give a correct version of this line of code. (3%)

b) Which bug causes the program to crash? Identify the line number where the bug appears and give a correct version of this line of code. (4%)

c) Which bug causes the program to produce incorrect output? Identify the line number where the bug appears and give a correct version of this line of code. (3%)

10. Write a C/C++ program to implement the Euclid's algorithm for the greatest common divisor of two numbers described by the flow chart below. (10%).



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