1. 20%

Let $y:\mathbb{R}\to\mathbb{R}$ be a C^∞ function that satisfies the differential equation $y''+y'-y=0$ for $x\in[0,L]$, where L is a positive constant . Suppose that $y(0)=y(L)=0$. Which of the following statements is true?

A. $y(x)=0$ for $0\le x\le L$

B. $y(x)>0$ for some $0<x<L$

C. $y(x)<0$ for some $0<x<L$.

Find and justify your answer.

2. 20%

Let k be a positive integer. For which values of the real number c does the differential equation

$$\frac{d^2x}{dt^2}-2c\frac{dx}{dt}+x=0$$

have a solution satisfying $x(0)=x(2\pi k)=0$?

3. 20%

Let $y:[0,1]\to\mathbb{R}$ be continuous, with $y(0)=y(1)=0$. Assume that y'' exists on $0<x<1$, with $y''+2y'+y\ge 0$.

Which of the following statements is true?

A. $y(x)>0$ for $0<x<1$

B. $y(x)\le 0$ for $0<x<1$

C. There exist $x_1>x_2>0$ such that $y(x_1)<0<y(x_2)$.

Find and justify your answer.

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4. 20%

Consider the equation $\frac{dy}{dx} = y - \sin y$ for $x \in \mathbb{R}$. Show that there exists $\varepsilon > 0$ such that if $|y_0| < \varepsilon$, then the solution $y = f(x)$ with $f(0) = y_0$ satisfies $\lim_{x \to -\infty} f(x) = 0$

5. 20%

i. Find a basis for the space of real solutions of the differential equation

(*) $$\sum_{n=0}^{7} \frac{d^n x}{dt^n} = 0$$

ii. Find a basis for the subspace of real solutions of (*) that satisfy $\lim_{t \to \infty} x(t) = 0$.

試題隨卷繳回