國立臺北科技大學 104 學年度碩士班招生考試

系所組別：3510 化學工程與生物科技系化學工程碩士班甲組

第三節　工程數學　試題

注意事項：

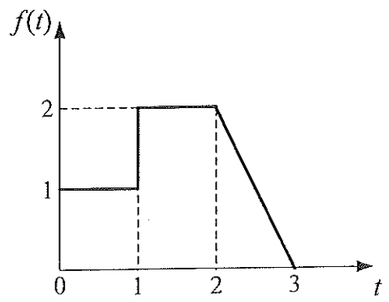
1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Find the general solution for y.

   (a) $\ln(y') = \ln(x) + y + 2$　(10%)

   (b) $\frac{dy}{dx} = \frac{e^{-x} - y}{x}$　(10%)

2. Find the Laplace transform and Fourier transform of the following function $f(t)$.　(20%)



3. Solve the following initial value problem for $y(t)$.　(15%)

   $y''(t) + 3y'(t) + 2y(t) = f(t)$, $y(0) = 0$, $y'(0) = 1$, and $f(t) = \begin{cases} 0, & 0 \le t < 2 \\ e^{-t}, & t \ge 2 \end{cases}$.

4. Let $\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$. Find a matrix $\mathbf{B}$ such that $\mathbf{B}^2 = \mathbf{A}^5 - 3\mathbf{A}^3 - 2\mathbf{A} + 5\mathbf{I}$, where $\mathbf{I}$ is a 2×2 identity matrix.　(10%)

5. (a) Find the unit outward normal vector of the surface $z = x^2 + y^2$ at the point $(0.5, 0.5, 0.5)$. (5%)

(b) If $\vec{F} = \frac{1}{3}x^3\vec{i} + \frac{1}{3}y^3\vec{j} + xy\vec{k}$ and $\vec{n}$ is the unit outward normal vector, evaluate $\iint_S (\vec{F} \cdot \vec{n}) dA$, where S is the entire surface of the region bounded by $z = x^2 + y^2$, $z \le 1$ and the plane $z = 1$. (10%)

6. (a) Solve the following partial differential equation: (15%)

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $0 \le x \le 5$, $t \ge 0$

with boundary conditions: $u(0,t) = u(5,t) = 1$ for $t > 0$

and initial condition: $u(x,0) = 1 + 10\sin(3\pi x/5)$ for $0 \le x \le 5$

(b) How long will it take for the maximum value of u on $0 \le x \le 5$ to be 2 ? (5%)