東吳大學 104 學年度碩士班研究生招生考試試題

系級	數學系碩士班 B 組（決策科學與海量資料分析）	考試時間	100 分鐘
科目	統計學	本科總分	100 分

1) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n. Using it, define the following: (15%)
   (a) Random Sample.
   (b) Sample mean.
   (c) Sample variance.
   (d) Sample standard deviation.
   (e) Sample Correlation Coefficient.

2) (a) What is meant by "Events $A_1, A_2, \ldots, A_n$ are mutually exclusive" ? (3%)

   (b) Let $A_1, A_2, \ldots, A_n$ be n mutually exclusive events such that $P(A_j) \neq 0$ for $j = 1, 2, \ldots, n$, and whose union is the entire sample space S. For any given event B such that $P(B) \neq 0$, state the Bayes' Law Formula for the posterior probabilities $P(A_j \mid B)$, $j = 1, 2, \ldots, n$. (6%)

   (c) The Graduate Entrance Examination (GRE) is a requirement for all applicants of MS programs. There are a variety of preparatory courses designed to help improve GRE scores. Let S be the set of all possible GRE scores, A_1 be the event that an applicant gets GRE score 650 or more, A_2 be the event that the applicant gets GRE score less than 650, and B be the event that the applicant take preparatory course. Suppose $P(A_1) = 0.40$, $P(B \mid A_1) = 0.20$ and $P(B \mid A_2) = 0.05$.
   (i) Are A_1 and A_2 mutually exclusive? (3%)
   (ii) Compute $P(A_1 \cap B)$ and $P(A_2 \cap B)$. (6%)
   (iii) Compute $P(B)$. (4%)
   (iv) Apply the Bayes' Law Formula to compute $P(A_1 \mid B)$ and $P(A_2 \mid B)$. (8%)

3) (a) Let Z be a standard normal random variable, i.e. $Z \sim N(0,1)$ and let F_Z be the (cumulative) distribution function of Z.
   (i) What is the probability density function of Z? (4%)
   (ii) If $F_Z(a) = 0.875$, what is the value of $F_Z(-a)$? (4%)
   (b) Let the random variable X have a Chi-Squared distribution with r degrees of freedom. What are the mean and the variance of X? (6%)

4) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a normal population with unknown mean μ and known variance $\sigma^2 > 0$.
   (a) Give an unbiased estimator for μ. (4%)
   (b) Find the maximum likelihood estimator of μ. (6%)
   (c) Give a confidence interval for μ with confidence coefficient α, $0 < \alpha < 1$. (6%)

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5) To test the null hypothesis H_0 against the alternative H_1 by using a test.

   (a) What is meant by the type I error of the test?　　　(4%)

   (b) What is meant by the type II error of the test?　　　(4%)

   (c) What is meant by the power of the test?　　　(4%)

6) (a) What is meant by a multinomial experiment?　　　(5%)

   (b) Describe the Chi-Squared Goodness-of-fit Test.　　　(8%)