東吳大學 104 學年度碩士班研究生招生考試試題

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1.(20%) Suppose that $f : S \to R$, $S \subset R^n$ and $\vec{a} \in S$. Suppose for any sequence $\{\vec{x}_k\}$ in S that converges to $\vec{a}$, the sequence $\{f(\vec{x}_k)\}$ converges to $f(\vec{a})$. Show that $f : S \to R^m$ is continuous at $\vec{a}$.

2.(20%) Let $f(x,y) = \begin{cases} y^2 & \text{if } x < 0 \\ y^2 & \text{if } y < 0 \\ -y^2 & \text{if } 0 \le y, 0 < x \end{cases}$.

For each (x,y) in the domain of f, find $\partial_1 f(x,y)$ and $\partial_2 f(x,y)$.

3. (20%) Suppose that f is three times differentiable on an open interval I containing a. Show that

$$\lim_{h\to 0} \frac{f(a+3h) - 3f(a+2h) + 3f(a+h) - f(a)}{h^3} = f'''(a).$$

4. (20%) Prove that $f(x) = \dfrac{1}{x^{2/3}+1}$ is uniformly continuous on R.

5.(20%) Prove that if f is integrable on $[0,1]$ and $\beta > 0$, then

$$\lim_{n\to\infty} n^{\alpha} \int_0^{1/n^{\beta}} f(x)dx = 0$$

for all $\alpha < \beta$.