國立中央大學104學年度碩士班考試入學試題

所別：統計研究所碩士班 不分組(一般生)　科目：數理統計　共 1 頁 第 1 頁
統計研究所碩士班 不分組(在職生)

本科考試可使用計算器，廠牌、功能不拘　　*請在答案卷（卡）內作答

參考用

1. Let $X_1,\ldots,X_n$ be i.i.d. random variables with p.d.f. given by

$$f_\lambda(x)=\lambda e^{-\lambda x},\quad \lambda>0,\quad x>0.$$

   1) [10%] Derive the moment generating function $M_{X_1}(t)$ of X_1.

   2) [10%] Derive the moment generating function $M_S(t)$ of $S=\sum_{i=1}^{n}X_i$.

   3) [10%] Write down the p.d.f. of $S=\sum_{i=1}^{n}X_i$.

2. Let $X_1,\ldots,X_n$ be i.i.d. random variables following a uniform distribution on $(0,\theta)$. Define $X_{(n)}=\max\{X_1,\ldots,X_n\}$.

   1) [10%] Derive an unbiased estimator of θ.
   2) [10%] Derive the maximum likelihood estimator (MLE) of θ.
   3) [10%] Prove the consistency of the MLE.
   4) [10%] $n(\theta-X_{(n)})$ converges *in distribution* to what distribution? Prove it.

3. Let $X_1,\ldots,X_n$ be i.i.d. random variables following $N(\mu,\sigma^2)$, where $\sigma^2>0$ is known.

   [15%] Derive a level α most powerful (MP) test for

$$H_0:\mu=\mu_0 \quad \text{vs.} \quad H_0:\mu=\mu_1>\mu_0.$$

   [15%] Derive a power function $\beta(\mu)$ of the above test.

   Use the notation: $\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\exp(-\frac{u^2}{2})du$.