考 試 科 目	微積分 8111.81161	所　別	應用數學系 8111	考試時間	3月1日(星期日)第一節

1. Evaluate the limits.

    (a) (6%) $\displaystyle \lim_{x \to 0} \frac{\sin^3(3x)(1 - \cos(2x))}{x \tan^4(5x)}$.

    (b) (6%) $\displaystyle \lim_{x \to \infty} \left(x - x^2 \ln\left(\frac{1+x}{x}\right) \right)$.

2. Evaluate the integrals.

    (a) (8%) $\displaystyle \int_0^1 \frac{1}{\sqrt{x(1-x)}} \, dx$.

    (b) (8%) $\displaystyle \int_0^4 \frac{\ln x}{\sqrt{x}} \, dx$.

    (c) (8%) $\displaystyle \int_0^\infty \frac{\sin x}{x} \, dx$.

    (d) (8%) $\displaystyle \int_0^1 \int_y^1 \sin(x^2) \, dx \, dy$.

3. Determine if each series converges or diverges.

    (a) (8%) $\displaystyle \sum_{n=1}^\infty \frac{n^{2n}}{(1 + 2n^2)^n}$.

    (b) (8%) $\displaystyle \sum_{n=1}^\infty \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$.

4. (10%) Evaluate the function $\varphi(t)$ defined by

$$\varphi(t) = \int_{-\infty}^\infty e^{-\frac{x^2}{2}} \cos(xt) \, dx.$$

5. (10%) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$.

6. (10%) Evaluate the line integral

$$\oint_C (3y - e^{\sin^3(2x)}) \, dx + (7x + \sqrt{y^4 + 3}) \, dy,$$

where C is the circle $x^2 + y^2 = 1$.

7. (10%) Suppose that the function $f : [a, b] \to \mathbb{R}$ is continuously differentiable and one to one on the interval $[a, b]$. Prove that

$$\int_a^b f(x) \, dx + \int_{f(a)}^{f(b)} f^{-1}(x) \, dx = bf(b) - af(a).$$