考試科目	微積分 421A	所別	企管所(乙組)	考試時間	2月28日(六)第三節

(All the calculations and justification process should be clear; otherwise, no point will be given.)

(1) Evaluate the limit.

(1a) (8%) $\lim_{x\to 0} \frac{\tan x}{x}$

(1b) (8%) $\lim_{x\to\infty} (1+\frac{2}{x})^x$

(2) Find dy/dx evaluated at the given point

(2a) (8%) $y(x)=\begin{cases} x^2\sin(\frac{1}{x}) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ at $x=0.$

(2b) (8%) $y=\sin(\cos x)$ at $x=0.$

(3) Evaluate the definite integral.

(3a) (8%) $\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx$

(3b) (8%) $\int_0^{\pi} t\cdot\sin t\;dt$

(3c) (8%) $\int_0^{\pi} \sin^4 t\;dt$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分 4412A	所別	企管所(乙組)	考試時間	2月28日(六)第三節

(4) Determine whether the series is convergent or divergent

(4a) (8%) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

(4b) (8%) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(5) Ella deposits \$10000 into an account in which interest accumulates at the rate of 4% per year, compounded continuously. She plans to withdraw \$2000 per year. This is modelled as the following differential equation

$$\frac{dQ(t)}{dt} = 0.04Q(t) - 2000.$$

(5a) (8%) Solve the differential equation.

(5b) (5%) How long does it take for her account to be exhausted? (You may use the facts: $\ln 5 = 1.6094$ and $\ln 2 = 0.6931$.)

(6) (15%) A manufacturer supplies refrigerators to two stores, A and B. The manager estimates that if x units are delivered to store A and y units to store B each month, the monthly profit will be $P(x, y)$ hundred dollars, where

$$P(x, y) = -0.02x^2 - 0.03xy - 0.05y^2 + 15x + 40y - 3000.$$

Each month, the company can produce exactly 700 refrigerators. How many refrigerators should be supplied to store A and how many to store B to maximize monthly profit?

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。