

國立高雄大學 104 學年度研究所碩士班招生考試試題

科目：微積分  
考試時間：100 分鐘

系所：統計學研究所  
本科原始成績：100 分

是否使用計算機：否

1. Evaluate the following limits:

(a) (10%)  $\lim_{x \rightarrow \infty} (1 - \frac{a}{x})^x$

(b) (10%)  $\lim_{x \rightarrow \infty} (\frac{1}{x} \ln \frac{1}{x})$

2. (10%) Find the second derivative of the function  $f(x) = \sqrt{x} \tan \sqrt{x}$ .

3. (10%) Find the 5th degree Taylor polynomials centered at 0 for the function  $f(x) = \sqrt{1+x^2}$ .

4. Evaluate the following integrals:

(a) (10%)  $\int \tan^2 x dx$

(b) (10%)  $\int x \ln x dx$

5. (15%) Use double integration to calculate the area of the region  $\Omega$  enclosed  $y = x^2$  and  $x + y = 2$ .

6. (10%) Determine whether  $\sum_{k=0}^{\infty} \frac{2^k + k^4}{3^k}$  convergence or divergence.

7. (15%) Find the area of the largest rectangle that can be inscribed in a circle of radius 5.