

國立臺灣師範大學 104 學年度碩士班招生考試試題

科目：工程數學

適用系所：電機工程學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (5 分) Find the general solution of $xy' + y = 4x^2y^2$.
2. (5 分) Find the general solution of $x^2 + y \cos(x) + y' \sin(x) = 0$.
3. (10 分) Find the general solution of $y'' - 4y' + 4y = \cos(x) + e^{2x}$.
4. (10 分) Find the general solution of $y'' - 2y' + 2y = e^x \sec(x)$.
5. (15 分) Find the power series solution of $y'' + x^2y = 0$ around $x = 0$ by Taylor's expansion, up to 3 nonzero terms.
6. (15 分) Use the Laplace transformation to find the solution of $y'' + 9y = f(x)$ with initial conditions, $y(0) = 1$ and $y'(0) = 3$. Note that the function $f(x)$ is defined in Fig. 1.

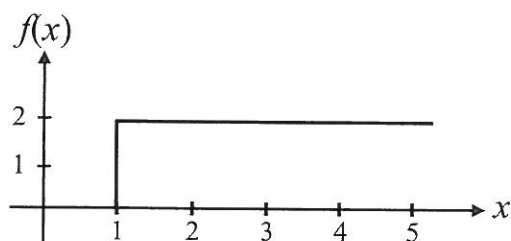


Fig. 1

7. Consider a periodic function $f(x)$ defined as follows:

$$f(x) = \begin{cases} -1, & -\pi \leq x < 0 \\ 1, & 0 \leq x < \pi \end{cases}, \text{ and } f(x + 2\pi) = f(x)$$

(a) (10 分) Find the Fourier series of $f(x)$.

(b) (5 分) Find the value of this series at $x = \pi$.

(c) (5 分) Use the result of (a) to determine $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$.

8. (10 分) Given a matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ with $P^{-1}AP = D$, where D is a diagonal matrix, find the matrix P .

9. (10 分) If $A = \begin{bmatrix} -3 & 2 \\ 4 & 4 \end{bmatrix}$, use Cayley-Hamilton theorem to find $A^4 + A^3 - 22A^2 - 38A + I$.