

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the integral equation and evaluate  $f(t)$  by using the method of

$$\text{Laplace transform: } f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau)e^{t-\tau}d\tau. (15\%)$$

2. Solve the initial-value problems:

$$y'' + 5y' + 4y = 2e^{-x}, y(0) = 2, y'(0) = 5. (15\%)$$

3. Solve the initial-value problems:

$$x^2 y'' + xy' - 4y = 0, y(1) = 0, y'(1) = 4. (15\%)$$

4. Determine the equation of tangent plane and normal line to the surface of

$$x^2 - 4y^2 + 4z^2 = 4 \text{ at point } (2,1,1). (15\%)$$

5. Solve the boundary value problem:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \pi, \quad t > 0,$$

$$\text{with boundary conditions: } u(0, t) = 0, \quad u(\pi, t) = 0, \quad t > 0$$

$$\text{and initial condition: } u(x, 0) = 100, \quad 0 < x < \pi.$$

(20%)

6. Find the eigenvalues and corresponding eigenvectors of the matrix:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}. (20\%)$$